

# 10.5 Binomial Expansion

## ~~Essential Question:~~

How can I find any term of a binomial expansion?

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$$(a + b)^0 = 1$$

$$(a + b)^1 = a + b$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(x+y)^{48}$$

## Binomial Expansion Theorem

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

a: first term in ( )  
 b: 2nd term in ( )  
 n: power of expansion  
 k: term #

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## Coefficients

Combination Notation & Formula

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

"n choose k"

ex.  $\binom{8}{5} = \frac{8!}{5!(8-5)!} = \frac{8!}{5! \cdot 3!} = \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 8 \cdot 7 = \boxed{56}$

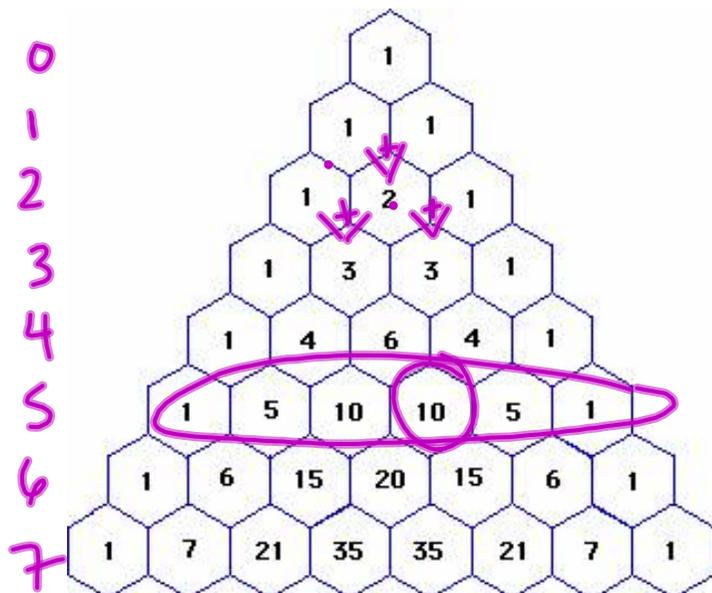
$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

! (factorial) - product of the integer and all integers before it

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## Coefficients & Pascal's Triangle



ex.  $\binom{5}{3}$

5th, 3rd #  
row  
= 10

## CALCULATOR

```

MATH NUM CPX PRB
1:rand
2:nPr
3:nCr ←
4:!
5:randInt(
6:randNorm(
7:randBin(
  
```

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## Exponents

→ power on a START @ n,  
goes down to 0

→ power on b STARTS @ 0,  
goes up to n

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

## Examples

1. Expand  $(x + 2)^5$

$$\binom{5}{0} x^5 (2)^0 + \binom{5}{1} x^4 (2)^1 + \binom{5}{2} x^3 (2)^2 + \binom{5}{3} x^2 (2)^3 + \binom{5}{4} x^1 (2)^4 + \binom{5}{5} x^0 (2)^5$$

$$x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

2. Find the 7th term of  $(x^2 + 2y)^{10}$

k = term - 1  
k = 6

$$\binom{10}{6} (x^2)^4 (2y)^6 = 210 x^8 (2^6) y^6 = 13440 x^8 y^6$$

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~~Other useful things...~~

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There is always one more term than the exponent of the expansion. ex.  $(x+y)^2 = x^2 + 2xy + y^2$   
3 terms

Exponents on the terms will ADD to the exponent of the expansion.

The exponent on the 2nd term will always match the bottom number of the combination.

Common combinations:  $\binom{n}{0} = \underline{1}$   $\binom{n}{n} = \underline{1}$   $\binom{n}{1} = \underline{n}$   $\binom{n}{n-1} = n$   $\binom{5}{4} = 5$