

# 10.5 Binomial Expansion

①  $(a+b)^6$

$$= \binom{6}{0} a^6 b^0 + \binom{6}{1} a^5 b^1 + \binom{6}{2} a^4 b^2 + \binom{6}{3} a^3 b^3 + \binom{6}{4} a^2 b^4 + \binom{6}{5} a^1 b^5 + \binom{6}{6} a^0 b^6$$

$$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

②  $(7+d)^4 = \binom{4}{0} 7^4 d^0 + \binom{4}{1} 7^3 d^1 + \binom{4}{2} 7^2 d^2 + \binom{4}{3} 7^1 d^3 + \binom{4}{4} 7^0 d^4$

$$= 2401 + 1372d + 294d^2 + 28d^3 + d^4$$

③  $(2b-7c)^3 = \binom{3}{0} (2b)^3 (-7c)^0 + \binom{3}{1} (2b)^2 (-7c)^1 + \binom{3}{2} (2b)^1 (-7c)^2 + \binom{3}{3} (2b)^0 (-7c)^3$

$$= 1 \cdot 8b^3 \cdot 1 + 3 \cdot 4b^2 \cdot (-7c) + 3 \cdot 2b \cdot 49c^2 + 1 \cdot 1 \cdot (-7)^3 c^3$$

$$= 8b^3 - 84b^2c + 294bc^2 - 343c^3$$

④  $(m-3)^{40} = \overset{1st}{\binom{40}{0} m^{40} (-3)^0} \quad \overset{2nd}{\binom{40}{1} m^{39} (-3)^1} \quad \overset{3rd}{\binom{40}{2} m^{38} (-3)^2}$

$$\boxed{1m^{40}} \quad \boxed{-120m^{39}} \quad \boxed{7020m^{38}}$$

⑤ 6th term of  $(c+d)^{12}$

$$\binom{12}{5} c^7 d^5 = \boxed{792c^7d^5}$$

match exponents add to total

⑥ 4th term of  $(3m - \frac{n}{3})^7$

$$\binom{7}{3} (3m)^4 \left(\frac{-n}{3}\right)^3$$

$$35(3^4)m^4 \left(\frac{-n^3}{3^3}\right) = 35(81)m^4 \left(\frac{-n^3}{27}\right) = \boxed{-105m^4n^3}$$

⑦  $(m + \frac{n^2}{2})^6$  7 terms total  
one more term than the exponent  
- middle term will be 4<sup>th</sup>

$$\binom{6}{3} m^3 \left(\frac{n^2}{2}\right)^3 = 20 m^3 \left(\frac{n^6}{8}\right) = \boxed{\frac{5}{2} m^3 n^6}$$

↑ one less than term #

⑧  $(4c + 2d)^6$  contains  $c^4$

$$\binom{6}{2} (4c)^4 (2d)^2 = 15 \cdot 256 c^4 \cdot 4 d^2 = \boxed{15360 c^4 d^2}$$

exp. add to 6