

# 12.6 Special Trig Limits

## Essential Question

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$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

ex.  $\lim_{y \rightarrow 0} \frac{\sin 4y}{4y}$  Argument matches top/bottom  
≠ 1  
 $x = 4y$

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## Limits Properties

$$\star \lim_{x \rightarrow c} 4f(x) = 4 \cdot \lim_{x \rightarrow c} f(x)$$

$$\text{ex. } \lim_{x \rightarrow 3} 7x = 7 \left( \lim_{x \rightarrow 3} x \right)$$

$7 \cdot 3 = 21$

$$\star \lim_{x \rightarrow c} f(x)g(x) = \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x)$$

$$\text{ex. } \lim_{x \rightarrow 9} x\sqrt{x} = \lim_{x \rightarrow 9} x \cdot \lim_{x \rightarrow 9} \sqrt{x}$$

$9 \cdot \sqrt{9} = 27$

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ex.  $\lim_{x \rightarrow 0} \frac{3 \sin 3x}{3x} = 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = 3 \cdot 1 = \boxed{3}$

multiply

ex.  $\lim_{x \rightarrow 0} \frac{\sin^2 3x}{\sin^2 7x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{1} \cdot \frac{\sin 3x}{1} \cdot \frac{1}{\sin 7x} \cdot \frac{1}{\sin 7x}$

$= \lim_{x \rightarrow 0} \frac{\sin 3x}{(3x)^1} \cdot \lim_{x \rightarrow 0} \frac{\sin 3x}{(3x)^1} \cdot \lim_{x \rightarrow 0} \frac{1}{\sin 7x} \cdot \lim_{x \rightarrow 0} \frac{1}{\sin 7x}$

$\lim_{x \rightarrow 0} 3x(1) \cdot 3x(1) \cdot \frac{1}{7x}(1) \cdot \frac{1}{7x}(1)$

$\lim_{x \rightarrow 0} \frac{9x^2}{49x^2} = \boxed{\frac{9}{49}}$

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$$\begin{aligned} \text{ex. } \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} &= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \sin x \\ &= 1 \cdot \sin 0 = 0 \end{aligned}$$

$$\begin{aligned} \text{ex. } \lim_{h \rightarrow 0} \frac{\sin 4h}{h^3} &= \lim_{h \rightarrow 0} 4 \frac{\sin 4h}{4h} \cdot \frac{1}{h^2} \\ &= \lim_{h \rightarrow 0} 4(1) \cdot \frac{1}{h^2} \\ &= \frac{4}{0} \quad \boxed{\text{DNE}} \end{aligned}$$

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ex.  $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x \sin 2x}$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x \sin 2x}$$

$$\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \cdot \frac{x \sin x}{x} \cdot \frac{1}{2x \sin 2x} \right)$$

$$\lim_{x \rightarrow 0} x \cdot \frac{1}{2x} = \lim_{x \rightarrow 0} \frac{1}{2} = \boxed{\frac{1}{2}}$$