

12.7 Intermediate Value Theorem

① $f(x) = x^3 + 2x - 1$ ← continuous since $f(x)$ is polynomial
zero on $[0, 1]$

$$\begin{aligned}f(0) &= 0^3 + 2(0) - 1 = -1 \\f(1) &= 1^3 + 2(1) - 1 = 2\end{aligned}$$

$-1 < 0 < 2$
 $f(0) < 0 < f(1) \quad \checkmark$

② $f(x) = \frac{x^2 + x}{x - 1}$ $\left[\frac{5}{2}, 4\right]$ $f(c) = 6$

$$f\left(\frac{5}{2}\right) = \frac{\left(\frac{5}{2}\right)^2 + \frac{5}{2}}{\frac{5}{2} - 1} = \frac{\frac{25}{4} + \frac{5}{2}}{\frac{5}{2} - \frac{2}{2}} = \frac{\frac{25}{4} + \frac{10}{4}}{\frac{3}{2}} = \frac{\frac{35}{4}}{\frac{3}{2}} = \frac{35}{4} \cdot \frac{2}{3} = \frac{35}{6} = 5.8$$

$$f(4) = \frac{4^2 + 4}{4 - 1} = \frac{20}{3} = 6.67 \quad 5.8 < 6 < 6.67 \quad \checkmark$$

$$\text{so } \frac{5}{2} < c < 4$$

③ $\cos x = x$ $\left[0, \frac{\pi}{2}\right]$ continuous since $\cos x$ & x are continuous
 $0 = x - \cos x$
solution \rightarrow zero

$$\begin{aligned}f(0) &= 0 - \cos 0 \\&= 0 - 1 \\&= -1\end{aligned}$$

$-1 < 0 < \frac{\pi}{2} \quad \checkmark$

$$\begin{aligned}f\left(\frac{\pi}{2}\right) &= \frac{\pi}{2} - \cos \frac{\pi}{2} \\&= \frac{\pi}{2} - 0\end{aligned}$$

④ $x^4 = 2^x$ (continuous ✓)

$$0 = 2^x - x^4$$

$$a = 1 \quad b = 2$$

$$-12 < 0 < 1 \quad \checkmark$$

$$f(1) = 2^1 - (1)^4 = 1$$

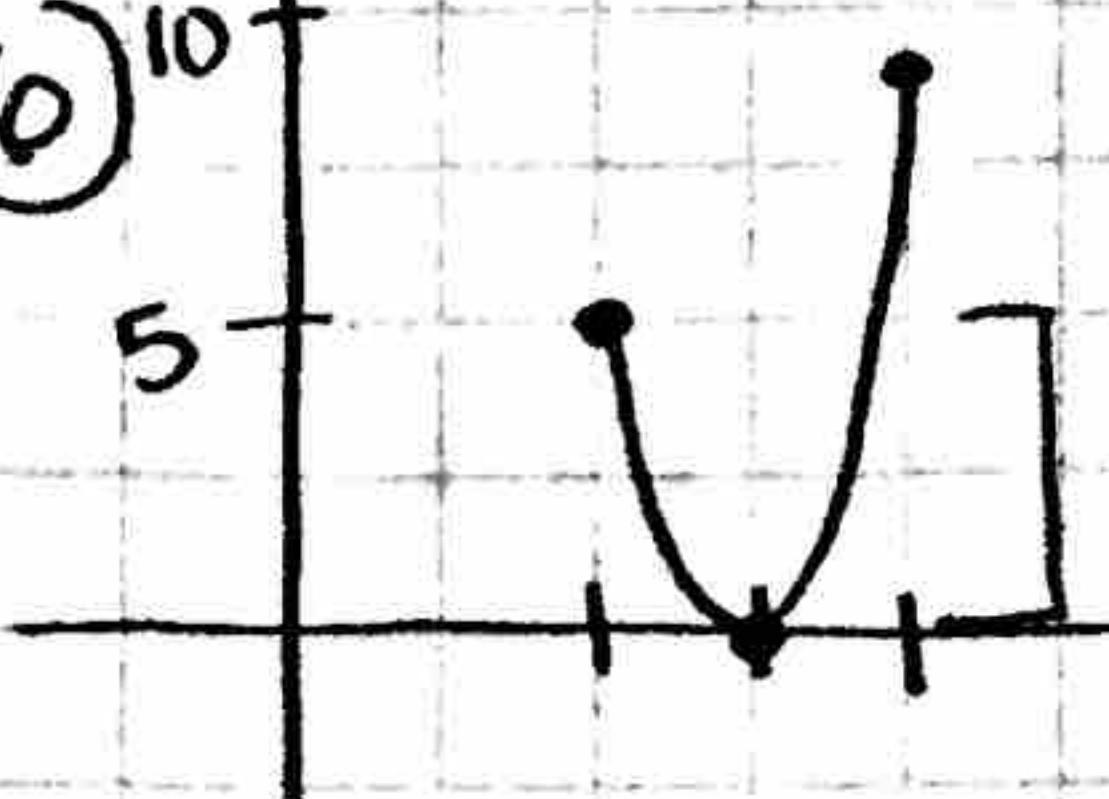
$$f(2) = 2^2 - (2)^4 = 4 - 16 = -12$$

$$\textcircled{5} \quad x^5 - 2x^4 - 11 = 0 \quad \text{solution} = 0$$

a) $f(-2) = 75$ NO b) $f(-1) = 14$ NO c) $f(0) = 11$ NO...
 $f(-1) = 14$ $f(0) = 11$ $f(1) = 12$

d) $f(1) = 12$ NO
 $f(2) = 11$ (really??) e) $f(2) = 11$
 $f(3) = -70$ $-70 < 0 < 11 \checkmark$

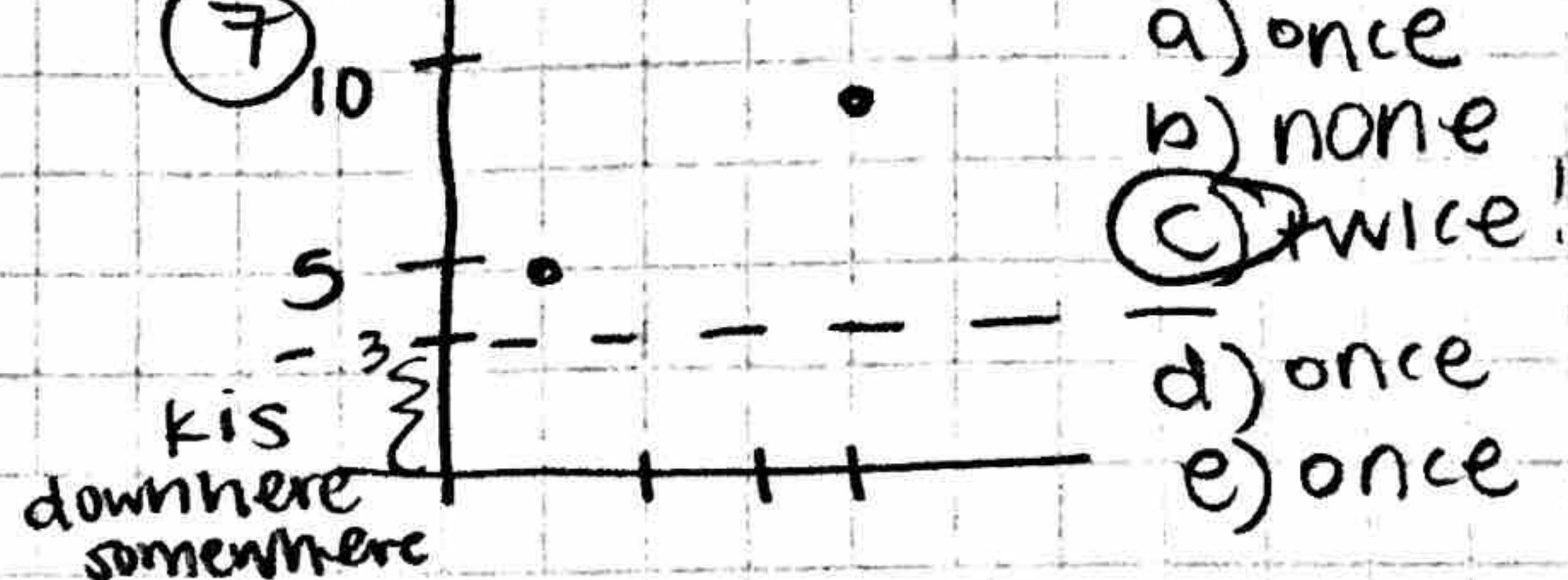
\textcircled{6}



- a) wouldn't touch at all
 b) only once
 c) twice! ☺
 d) only once
 e) only once

k is a y value,
 function needs
 to touch $y=k$ twice

\textcircled{7}



- a) once
 b) none
 c) twice!
 d) once
 e) once

2 solutions, horizontal
 line would touch twice

\textcircled{8} $f(x) = \begin{cases} x^2 - 5 & \text{for } x < 0 \\ 3 & \text{for } x = 0 \\ x^2 + 5 & \text{for } x > 0 \end{cases}$

a) $\lim_{x \rightarrow 0^+} f(x) = \boxed{5}$
 $0^2 + 5$

b) $\lim_{x \rightarrow 0^-} f(x) = \boxed{-5}$
 $0^2 - 5$

c) $\lim_{x \rightarrow 3} f(x) = \boxed{14}$
 $3^2 + 5$

d) $x=0$ (jump)

e) yes, but not
 vice versa.

(limit existing doesn't
 guarantee continuity)

\textcircled{9} $f(x) = \begin{cases} x + c & x < -2 \\ cx^2 + 7 & x \geq -2 \end{cases}$

$$\begin{aligned} -2 + c &= c(-2)^2 + 7 \\ -2 + c &= 4c + 7 \\ -9 &= 3c \end{aligned}$$

$$\boxed{-3 = c}$$