Name:_____

Intermediate Value Theorem

If f is continuous on $[a,b]$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$

1. Use the Intermediate Value Theorem to show that $f(x)=x^3+2x-1$ has a zero on $[0,1]$.

2. Verify that the IVT applies to the interval and find the value of c guaranteed by the theorem.

$f(x)=\frac{x^2+x}{x-1}$ $\left[\frac{5}{2},4\right]$ $f(c)=6$

3. Use the IVT to show that there exists a solution to cos(x) = x on the interval [0, $\frac{\pi}{2}$]

4. Use the IVT to show that the equation $x^4=2^x$ has at least one solution (you need to choose an a and b value on your own)

5. By applying the Intermediate Value Theorem choose the interval over which $x^5 = 2x^4 + 11$ will have a solution.

a) $[-2, -1]$ b) $[-1, 0]$ c) $[0, 1]$

d) $[1, 2]$ e) $[2, 3]$

6. Let f be a continuous function on $[2, 4]$ and have the values shown.

The equation $f(x) = k$ must have at least 2 solutions on $[2, 4]$ for which value(s) of k?

x	2	3	4
$f(x)$	5	0	9

a) $k > 9$

b) 7

c) $0 < k < 5$

d) $k > 5$

e) $5 < k < 9$

7. f is continuous on $[2, 4]$ and has the values shown.

The equation $f(x) = 3$ must have at least 2 solutions on $[2, 4]$ for $k =$ _____.

a) 3 b) 4 c) 2

d) 5 e) 6

x	2	3	4
$f(x)$	5	k	9

8. Consider $f(x) = \begin{cases} x^2 - 5 & \text{for } x < 0, \\ 3 & \text{for } x = 0, \\ x^2 + 5 & \text{for } x > 0 \end{cases}$

a) $\lim_{x \to 0^+} f(x) =$ _____

b) $\lim_{x \to 0^-} f(x) =$ _____

c) $\lim_{x \to 3} f(x) =$ _____

d) Where is $f(x)$ discontinuous?

e) If a function is continuous at $x = a$, does this necessarily mean that $\lim_{x \to a}$ exists?

9. Consider $f(x) = \begin{cases} x + c & \text{for } x < -2, \\ cx^2 + 7 & \text{for } x \geq -2 \end{cases}$

For what value of the constant c is f continuous for all real numbers?