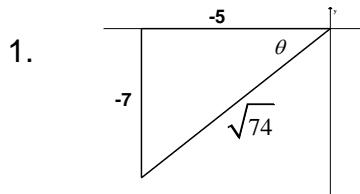


Pre-AP PreCalculus Fall Semester Exam Review

SOLUTIONS



$$\begin{aligned}\sin \theta &= -\frac{7\sqrt{74}}{74} & \csc \theta &= -\frac{\sqrt{74}}{7} \\ \cos \theta &= -\frac{5\sqrt{74}}{74} & \sec \theta &= -\frac{\sqrt{74}}{5} \\ \tan \theta &= \frac{7}{5} & \cot \theta &= \frac{5}{7}\end{aligned}$$

2. $\cos \frac{\pi}{4} \sin \frac{7\pi}{6} - \sin \frac{\pi}{6} \cos \frac{3\pi}{4}$

$$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left[-\frac{\sqrt{2}}{2}\right]$$

$$-\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} = 0$$

3. $\cot^2 \frac{11\pi}{6} - \csc^2 \frac{11\pi}{6}$

$$\left(-\sqrt{3}\right)^2 - (-2)^2$$

$$3 - 4 = -1$$

4. $\sin \frac{\pi}{2} + 6 \cos \frac{\pi}{3} = (1) + 6\left(\frac{1}{2}\right) = 1 + 3 = 4$

5. $\frac{\cos \frac{5\pi}{3}}{\sin \frac{5\pi}{3}} = \cot \frac{5\pi}{3} = -\frac{\sqrt{3}}{3}$

6. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6} + \tan^2 \frac{\pi}{6} - \sec^2 \frac{\pi}{6}$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2$$

$$\frac{1}{4} + \frac{3}{4} + \frac{1}{3} - \frac{4}{3} = \frac{4}{4} - \frac{3}{3} = 0$$

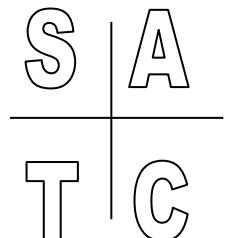
7. $40^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{9}$

8. $\frac{\pi}{9} \cdot \frac{180^\circ}{\pi} = 20^\circ$

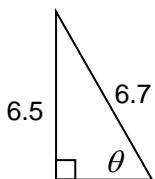
9. Positive: $84^\circ + 360^\circ = 444^\circ$

Negative: $84^\circ - 360^\circ = -276^\circ$

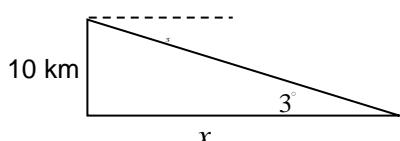
10. Q IV



11.



12.



$\sin \theta = \frac{6.5}{6.7}$

$\theta = \sin^{-1}\left(\frac{6.5}{6.7}\right)$

$\theta \approx 76^\circ$

a) $\tan 3 = \frac{10}{x}$

$x = \frac{10}{\tan 3}$

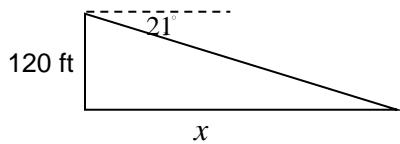
$x \approx 190.81 \text{ km}$

b) $\tan \theta = \frac{10}{300}$

$\theta = \tan^{-1}\left(\frac{10}{300}\right)$

$\theta \approx 2^\circ$

13.



$$\sin 21^\circ = \frac{120}{x}$$

$$x = \frac{120}{\sin 21^\circ}$$

$$x \approx 334.85 \text{ ft}$$

Remember:

$$y = C + A \sin B(x - D) \quad \text{where}$$

 A = amplitude C = vertical displacement (shift) D = horizontal displacement (shift)

$$\text{Period} = \frac{360}{B} = \frac{2\pi}{B} \text{ for sine or cosine}$$

$$\text{Period} = \frac{180}{B} = \frac{\pi}{B} \text{ for tangent}$$

Critical Points occur every $\frac{\text{Period}}{4}$

14. $y = -10 + 20 \sin 2\left(x - \frac{\pi}{8}\right)$

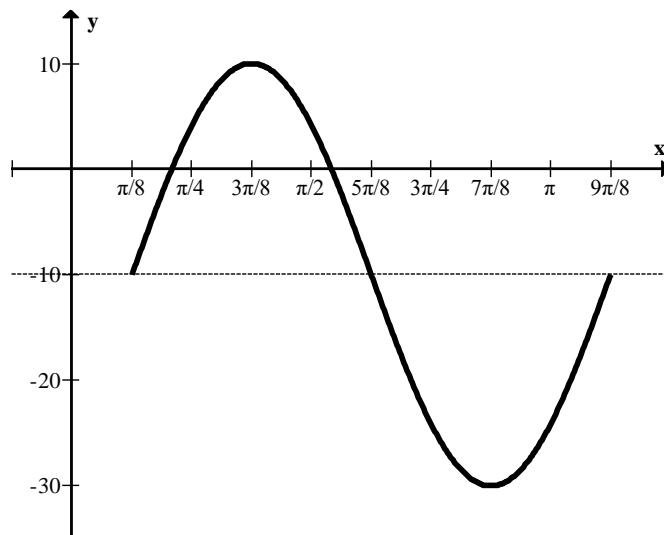
$$C = -10$$

$$A = 20$$

$$D = \frac{\pi}{8} \quad (\text{middle pt } @ \frac{\pi}{8})$$

$$\text{Period} = \frac{2\pi}{2} = \pi$$

$$\text{Crit. Pnts} = \frac{\pi}{4} = \frac{2\pi}{8}$$



15. $y = -5 \cos \frac{1}{2}(x + \pi) + 3$

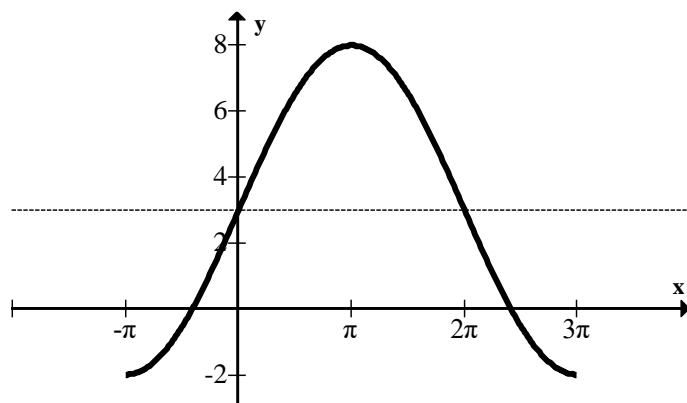
$$C = 3$$

$$A = 5$$

$$D = -\pi \quad (\text{Low point})$$

$$\text{Period} = \frac{2\pi}{1/2} = 4\pi$$

$$\text{Crit. Pnts} = \frac{4\pi}{4} = \pi$$



16. $y = 3 + 2\cos \frac{1}{5}(x - \pi)$

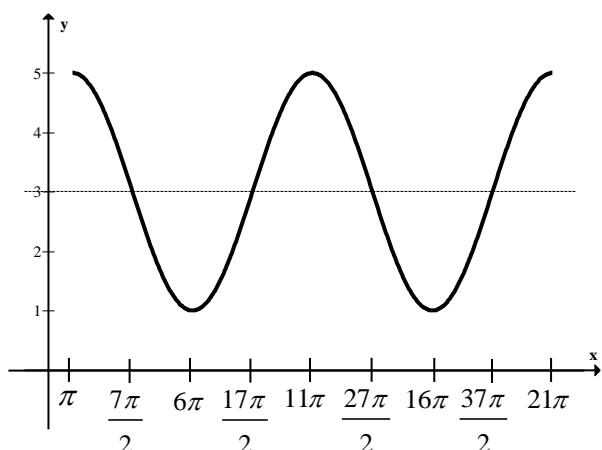
A = 2

C = 3

D = π (high point)

Period = $\frac{2\pi}{\frac{1}{5}} = 10\pi$

Crit. Pnts = $\frac{10\pi}{4} = \frac{5\pi}{2}$



17. $y = 2 - 6\sin \frac{\pi}{4}(x - 1)$

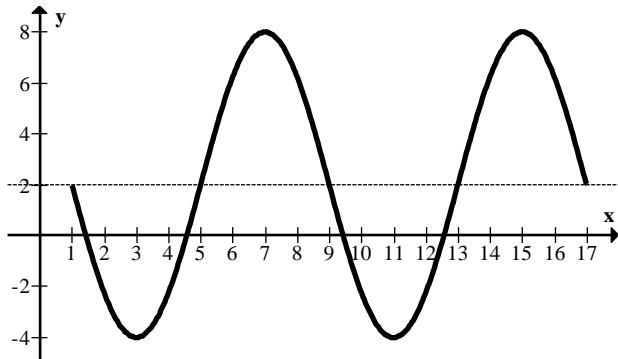
A = 6

C = 2

D = 1 (middle going down)

Period = $\frac{2\pi}{\frac{\pi}{4}} = 8$

Crit. Pnts = $\frac{8}{4} = 2$



18. $y = 1 + \sin 4\left(x + \frac{\pi}{4}\right)$

To find B $B = \frac{2\pi}{\text{Period}} = \frac{2\pi}{\frac{\pi}{2}} = 4$

$y = 1 - \sin 4(x)$

$y = 1 + \cos 4\left(x + \frac{\pi}{8}\right)$

$y = 1 - \cos 4\left(x - \frac{\pi}{8}\right)$

19. $y = -2 + 5 \sin \frac{\pi}{15}(x + 12.5)$

To find B $B = \frac{2\pi}{\text{Period}} = \frac{2\pi}{30} = \frac{\pi}{15}$

$$y = -2 - 5 \sin \frac{\pi}{15}(x - 2.5)$$

$$y = -2 + 5 \cos \frac{\pi}{15}(x + 5)$$

$$y = -2 - 5 \cos \frac{\pi}{15}(x - 10)$$

20. $y = 2 - 2 \sin \frac{\pi}{4}(x - 3)$

To find B

$$y = 2 + 2 \cos \frac{\pi}{4}(x - 1)$$

$$\frac{1}{2} \text{Period} = 4$$

$$\text{Period} = 8$$

$$B = \frac{2\pi}{\text{Period}} = \frac{2\pi}{8} = \frac{\pi}{4}$$

21. $f(x) = 5 + 2 \cos \frac{\pi}{4}(x - 10)$

a) $f(17.3) = 5 + 2 \cos \frac{\pi}{4}(17.3 - 10)$

$$f(17.3) \approx 6.705$$

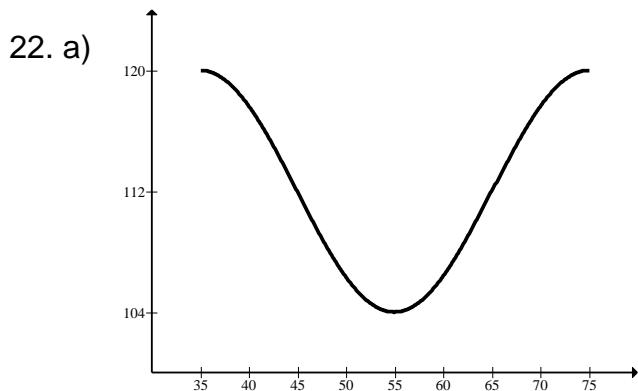
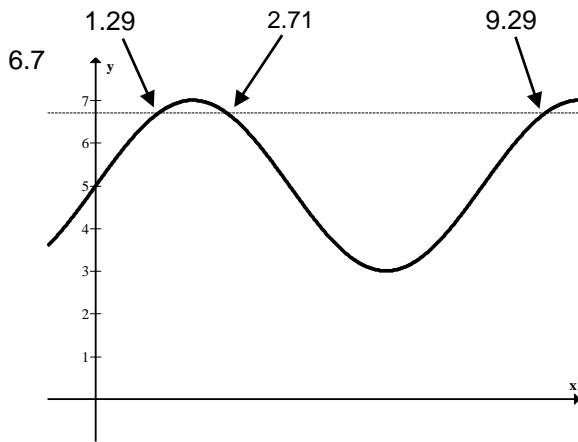
$$f(x) = 6.7$$

b) $6.7 = 5 + 2 \cos \frac{\pi}{4}(x - 10)$

$$x = 1.29, 2.71, 9.29$$

c) Maximum y-value is 7

$$x = 2$$

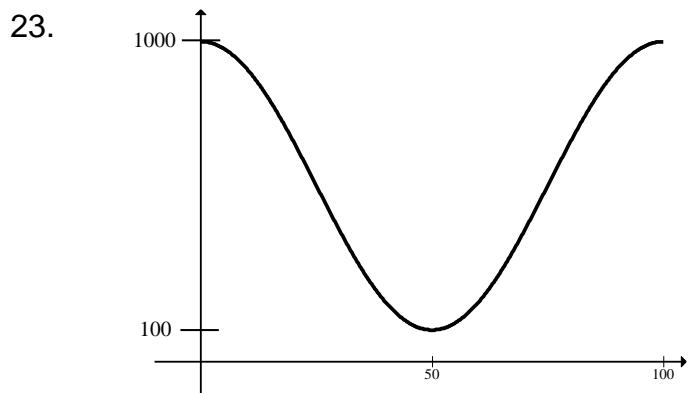


b) $y = 112 + 8 \cos \frac{\pi}{20}(x - 35)$

c) $y(0) \approx 118^\circ$

d) $114 = 112 + 8 \cos \frac{\pi}{20}(x - 35)$

$$x = 3.39, 26.61, 43.39$$

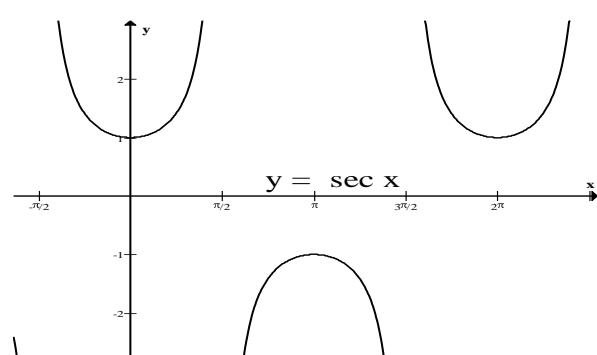
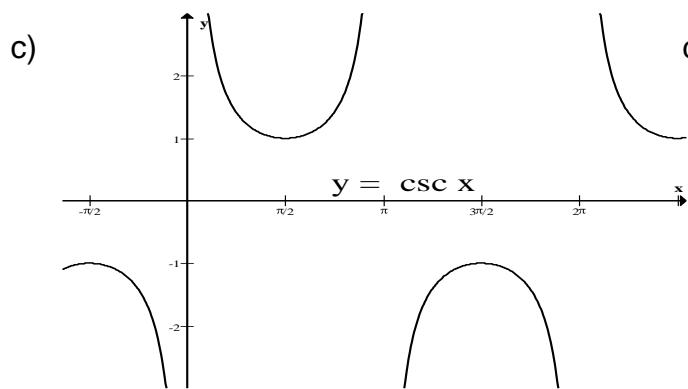
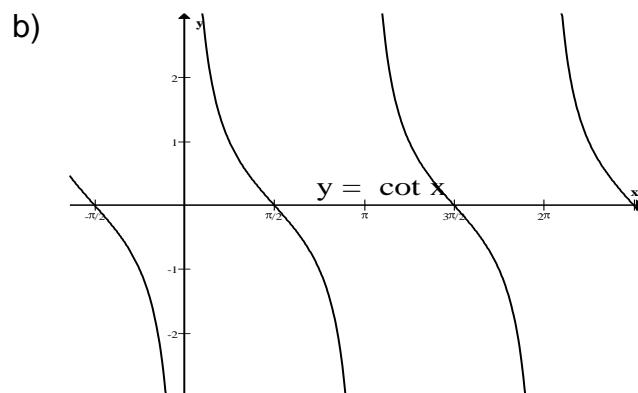
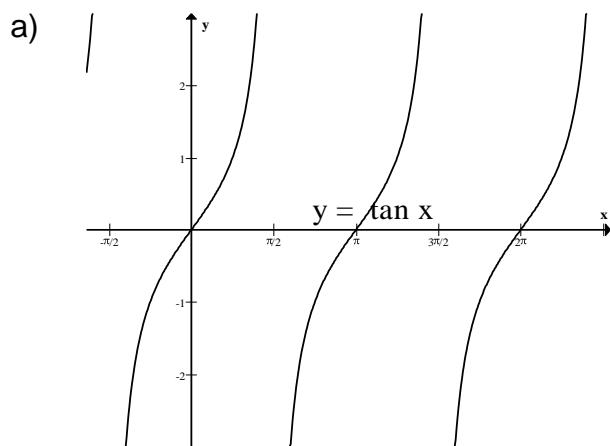


a) $d = 550 + 450 \cos \frac{\pi}{50} t$

b) $t = \frac{\pi}{50} \cos^{-1} \left(\frac{d - 550}{450} \right)$

c) $700 = 550 + 450 \cos \frac{\pi}{50} t$
 $t = 19.59, 80.41$
 $80.41 - 19.59 = 60.82 \text{ minutes}$

24.



25. $x = \cos^{-1} \left(-\frac{1}{\sqrt{2}} \right)$
 $\frac{3\pi}{4} \text{ and } \frac{5\pi}{4}$

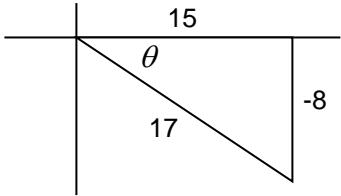
27. $x = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$
 $\frac{\pi}{6} \text{ and } \frac{7\pi}{6}$

28. $x = \cos^{-1}(-1) = \pi$

29. Principal inverse value, negative ratio for sine in QIV

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

30. $\cos\left(\arcsin\left(-\frac{8}{17}\right)\right) = \frac{15}{17}$



31. $\cos^{-1}\left(\sin\frac{7\pi}{6}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

32.

$$\frac{1}{1+\cos x} = \csc^2 x - \csc x \cot x$$

$$\left(\frac{1-\cos x}{1-\cos x}\right)\left(\frac{1}{1+\cos x}\right) =$$

$$\frac{1-\cos x}{1-\cos^2 x} =$$

$$\frac{1-\cos x}{\sin^2 x} =$$

$$\frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x} =$$

$$\frac{1}{\sin^2 x} - \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} =$$

$$\csc^2 x - \csc x \cot x = \csc^2 x - \csc x \cot x$$

34. $\frac{1-3\cos x-4\cos^2 x}{\sin^2 x} = \frac{1-4\cos x}{1-\cos x}$

$$\frac{(1-4\cos x)(1+\cos x)}{1-\cos^2 x} =$$

$$\frac{(1-4\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x)} =$$

$$\frac{1-4\cos x}{1-\cos x} = \frac{1-4\cos x}{1-\cos x}$$

33.

$$\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x} = 2\csc x$$

$$\left(\frac{1+\cos x}{1+\cos x}\right)\left(\frac{\sin x}{1-\cos x}\right) + \frac{1-\cos x}{\sin x} =$$

$$\frac{(1+\cos x)\sin x}{1-\cos^2 x} + \frac{1-\cos x}{\sin x} =$$

$$\frac{(1+\cos x)\sin x}{\sin^2 x} + \frac{1-\cos x}{\sin x} =$$

$$\frac{1+\cos x}{\sin x} \cdot \frac{\sin x}{\sin x} + \frac{1-\cos x}{\sin x} =$$

$$\frac{1+\cos x}{\sin x} + \frac{1-\cos x}{\sin x} =$$

$$\frac{2}{\sin x} =$$

$$2\csc x = 2\csc x$$

35. $\tan x = \frac{1-\cos 2x}{\sin 2x}$

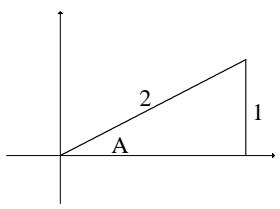
$$= \frac{1-(1-2\sin^2 x)}{2\sin x \cos x}$$

$$= \frac{1-1+2\sin^2 x}{2\sin x \cos x}$$

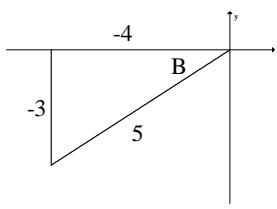
$$= \frac{2\sin^2 x}{2\cos x \sin x}$$

$$= \frac{\sin x}{\cos x} \\ = \tan x$$

36. $\sin A = \frac{1}{2}$
 $\cos A > 0$

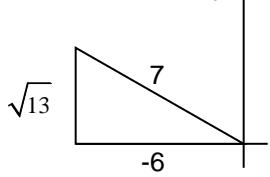


$\tan B = \frac{3}{4}$
 $\sin B < 0$



$$\begin{aligned}\sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \left(\frac{1}{2}\right)\left(-\frac{4}{5}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{3}{5}\right) \\ &= -\frac{4}{10} - \frac{3\sqrt{3}}{10} \\ &= \frac{-4-3\sqrt{3}}{10}\end{aligned}$$

37. $\cos A = -\frac{6}{7}$



Quadrant II $\sin 2A = 2 \sin A \cos A$

$$\begin{aligned}&= 2\left(\frac{\sqrt{13}}{7}\right)\left(-\frac{6}{7}\right) \\ &= -\frac{12\sqrt{13}}{49}\end{aligned}$$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\begin{aligned}&= \left(-\frac{6}{7}\right)^2 - \left(\frac{\sqrt{13}}{7}\right)^2 \\ &= \frac{36}{49} - \frac{13}{49} \\ &= \frac{23}{49}\end{aligned}$$

$$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{-\frac{12\sqrt{13}}{49}}{\frac{23}{49}} = -\frac{12\sqrt{13}}{23}$$

38. $\cos \theta = -\frac{3}{5}$, $180^\circ < \theta < 270^\circ \Rightarrow 90^\circ < \frac{1}{2}\theta < 135^\circ \Rightarrow \frac{1}{2}\theta$ is in Quadrant II

$$\sin \frac{1}{2}\theta = +\sqrt{\frac{1}{2}(1-\cos \theta)} = \sqrt{\frac{1}{2}\left(1-\frac{-3}{5}\right)} = \sqrt{\frac{1}{2}\left(\frac{8}{5}\right)} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \frac{1}{2}\theta = -\sqrt{\frac{1}{2}(1+\cos \theta)} = -\sqrt{\frac{1}{2}\left(1+\frac{-3}{5}\right)} = -\sqrt{\frac{1}{2}\left(\frac{2}{5}\right)} = -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\tan \frac{1}{2}\theta = \frac{\sin \frac{1}{2}\theta}{\cos \frac{1}{2}\theta} = \frac{\frac{2\sqrt{5}}{5}}{-\frac{\sqrt{5}}{5}} = -\frac{2\sqrt{5}}{\sqrt{5}} = -2$$

39. $2\sin\theta\cos\theta = \sqrt{2}\cos\theta$ $\theta \in \{\text{real numbers of degrees}\}$

$$2\sin\theta\cos\theta - \sqrt{2}\cos\theta = 0$$

$$\cos\theta(2\sin\theta - \sqrt{2}) = 0$$

$$\cos\theta = 0$$

$$2\sin\theta - \sqrt{2} = 0$$

$$\theta = \cos^{-1} 0$$

$$\sin\theta = \frac{\sqrt{2}}{2}$$

$$\theta = \pm 90 + 360n$$

$$\theta = \sin^{-1} \frac{\sqrt{2}}{2}$$

$$\theta = 45 + 360n$$

$$\theta = 135 + 360n$$

40. $2\cos^2 x - 5\cos x + 2 = 0$ $x \in [0, 2\pi)$

$$(2\cos x - 1)(\cos x - 2) = 0$$

$$2\cos x - 1 = 0$$

$$\cos x - 2 = 0$$

$$x = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

$$\cos x = \frac{1}{2}$$

$$\cos x = 2$$

$$x = \cos^{-1} 2$$

$$x = \cos^{-1} \frac{1}{2}$$

$$\emptyset$$

$$x = \pm \frac{\pi}{3} + 2\pi n$$

41.

$$4\sin x \cos x = \sqrt{3}$$

$$2x = \frac{\pi}{3} + 2\pi n$$

$$2x = \left(\pi - \frac{\pi}{3}\right) + 2\pi n$$

$$x = \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3} \right\}$$

$$2\sin x \cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6} + \pi n$$

$$2x = \frac{2\pi}{3} + 2\pi n$$

$$\sin 2x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} + \pi n$$

$$2x = \sin^{-1} \frac{\sqrt{3}}{2}$$

42. $\sin 2\theta \cos 64^\circ + \cos 2\theta \sin 64^\circ = \frac{\sqrt{3}}{2}$ $x \in [0, 360)$

$$\sin(2\theta + 64) = \frac{\sqrt{3}}{2}$$

$$2\theta + 64 = \sin^{-1} \frac{\sqrt{3}}{2}$$

$$2\theta + 64 = 60 + 360n \quad 2\theta + 64 = (180 - 60) + 360n \quad \theta = \{28^\circ, 178^\circ, 208^\circ, 358^\circ\}$$

$$2\theta = -4 + 360n \quad 2\theta + 64 = 120 + 360n$$

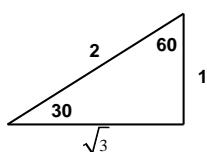
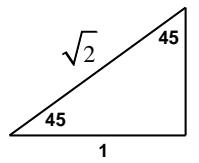
$$\theta = -2 + 180n \quad 2\theta = 56 + 360n$$

$$\theta = 28 + 180n$$

43. $\sin^2 x + 2 \sin x = 3$ $\sin x - 1 = 0$ $\sin x + 3 = 0$
 $\sin^2 x + 2 \sin x - 3 = 0$ $\sin x = 1$ $\sin x = -3$
 $(\sin x - 1)(\sin x + 3) = 0$ $x = \sin^{-1}(1)$ $x = \sin^{-1}(-3)$ $x = \left\{ \frac{\pi}{2} \right\}$
 $\frac{\pi}{2}$ DNE

44. $\tan^2 x = 1$ $x = \tan^{-1}(1)$ $x = \tan^{-1}(-1)$ $x = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$
 $\sqrt{\tan^2 x} = \sqrt{1}$ $\frac{\pi}{4}, \frac{5\pi}{4}$ $\frac{3\pi}{4}, \frac{7\pi}{4}$

Good To Know



S	A
T	C

	S	C	T
0	0	1	0
90	1	0	U
180	0	-1	0
270	-1	0	U

Restricted Ranges:

sine, cosecant, tangent $-\frac{\pi}{2}, \frac{\pi}{2}$ (Positive – QI, Negative – negative ref. angle)

cosine, secant, cotangent $0, \pi$ (Positive – QI, Negative – QII)