Name: $\qquad$

## Sinusoidal Modeling Project

This project will count as a TEST grade and will be due at the beginning of class Friday, Sept. 23

## Part 1: Modeling the Motion of a Pendulum

As a simple pendulum swings back and forth, its displacement can be modeled using a standard sinusoidal equation of the form:

$$
y=a \cos (b(x-d))+c
$$

where y represent the pendulum's distance from a fixed point and $x$ represents the total elapsed time in seconds. In this project, you will use a motion detection device called a CBR (Calculator Based Ranger) to collect distance and time data for a swinging pendulum, then find a mathematical model that describes the pendulum's motions.

## Materials

1 meter of string
ball fastened to the end of the string graphing calculator
calculator to CBR connection cable
CBR

## Collecting the Data

Once the CBR is set up to collect time and distance readings for 2 seconds, start the pendulum swinging in front of the detector. Make sure only the string and washer/ball are in front of the detector and there is nothing in the path that would interrupt the data. Once you start the pendulum swinging in front of the detector, then activate the system.
Even though your calculator will gather the data for your scatter plot and regression equation, go ahead and record your data here .

| Total elapsed <br> time (seconds) | Distance from the <br> CBR (meters) |  | Total elapsed <br> time (seconds) | Distance from the <br> CBR (meters) | Total elapsed <br> time (seconds) | Distance from the <br> CBR (meters) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  | 0.7 |  | 1.4 |  |  |
| 0.1 |  | 0.8 |  | 1.5 |  |  |
| 0.2 |  | 0.9 |  | 1.6 |  |  |
| 0.3 |  | 1.0 |  | 1.7 |  |  |
| 0.4 |  | 1.1 |  | 1.8 |  |  |
| 0.5 |  | 1.2 |  | 1.9 |  |  |
| 0.6 |  | 1.3 |  | 2.0 |  |  |

1. Find a sinusoidal regression equation to model the data above (using your calculator)

The data table below shows a sample set of data collected as a pendulum swung back and forth in front of a CBR.

| Total elapsed <br> time (seconds) | Distance from the CBR <br> (meters) |
| :---: | :---: |
| 0 | 0.665 |
| 0.1 | 0.756 |
| 0.2 | 0.855 |
| 0.3 | 0.903 |
| 0.4 | 0.927 |
| 0.5 | 0.931 |
| 0.6 | 0.897 |
| 0.7 | 0.837 |
| 0.8 | 0.753 |
| 0.9 | 0.663 |
| 1.0 | 0.582 |
| 1.1 | 0.525 |
| 1.2 | 0.509 |
| 1.3 | 0.495 |
| 1.4 | 0.521 |
| 1.5 | 0.575 |
| 1.6 | 0.653 |
| 1.7 | 0.741 |
| 1.8 | 0.825 |
| 1.9 | 0.888 |
| 2.0 | 0.921 |

On your calculator, graph a scatter plot for the data above. Make sure you find a window that fits this data.
2. Without using regression on your calculator, use the data above to find values for $a, b, c$ and $d$ so that the equation $\mathrm{y}=a \cos (b(x-d))+c$ fits the distance vs time data plot. You are not finding a sinusoidal regression yet. You are using the data above and your scatter plot to come up with the cosine equation that models the data. This equation should be different from your equation on the first page because the data is different.
3. What are the physical meanings for the constants $a$ and $k$ in the modeling equation $y=a \cos (b(x-d))+c$ ? (what do they represent specifically in this problem?)
4. Which, if any, of the values of $a, b, c$ and/or $d$ would change if you used the equation $y=a \sin (b(x-d))+c$ to model the data set?
5. Use your calculator to find a sinusoidal regression equation to model this data set. Notice the calculator's regression equation has not factored out "b." Factor out b for your final sine equation below. Round a, b, c and d to three decimal places.
6. Graph the cosine equation from \#2 here. Make sure to number and label the x and y axes.

Using a red colored pencil, show the amplitude, a, on the graph.
Using a blue colored pencil, show the period of the function. Also show the calculation here for the period using the $b$ value.
Using a green colored pencil, show the horizontal shift, d.
Using a black colored pencil, show the vertical shift, c.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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7. Compare your equation from the given data (\#2 and \#5) to the equation you got from recording data with the CBR (\#1). Which parts of the equation were different? Why were they different?

Part 2: Where in the World is Carmen Sinusoidal

1. Choose 6 cities from different continents around the globe. Try to vary the cities' locations by choosing different hemispheres, latitudes, longitudes, altitudes, and proximities to things like large bodies of water, deserts, mountain ranges, etc. Mark the location of each on the map below

2. Find the average monthly temperature for each city and record below. Some good sites to search include www.climate-zone.com, or www.weatherbase.com. Make sure all temperatures are in Fahrenheit.
3. Go to www.desmos.com and create a scatterplot (add item, table) for each of your cities in a different color (click the settings wheel to change the color). Include values for a second year to see the sinusoidal pattern more clearly ( 13 corresponds to January year 2, 14 is February year 2, etc. through 24)
4. For your European, North and South American cities, create a cosine equation without using regression. Include this on your Desmos graph. You can hide your other scatterplots to see one at a time by clicking the color circle.
5. For your African, Asian, and Australian cities, create a sin equation using regression on your calculator. Include these on your Desmos graph. You can hide your other graphs and scatter plots by clicking on the color wheel.
6. Submit your Desmos graph to svkorotkow@g.risd.org and answer the questions below.

North America City:_Color on Desmos:

| Month | 1 <br> (Jan) | 2 <br> (Feb) | 3 <br> (Mar) | 4 <br> (April) | 5 <br> (May) | 6 <br> (June) | 7 <br> (July) | 8 <br> (Aug) | 9 <br> (Sept) | 10 <br> (Oct) | 11 <br> (Nov) | 12 <br> (Dec) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Avg <br> Temp |  |  |  |  |  |  |  |  |  |  |  |  |

Cosine equation (don't use regression):

| South America City: |  |  |  |  |  |  |  | Color on Desmos: |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Month | $\begin{array}{\|l\|} \hline 1 \\ \text { (Jan) } \end{array}$ | $\begin{array}{\|l\|} \hline 2 \\ \text { (Feb) } \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 3 \\ \text { (Mar) } \\ \hline \end{array}$ | $\begin{aligned} & 4 \\ & \text { (April) } \end{aligned}$ | $\begin{aligned} & 5 \\ & \text { (May) } \end{aligned}$ | 6 <br> (June) | $\begin{array}{\|l\|} \hline 7 \\ \text { (July) } \end{array}$ | 8 <br> (Aug) | $\begin{aligned} & 9 \\ & \text { (Sept) } \end{aligned}$ | $\begin{aligned} & \hline 10 \\ & (\mathrm{Oct}) \end{aligned}$ | $\begin{aligned} & 11 \\ & \text { (Nov) } \\ & \hline \end{aligned}$ | $\begin{aligned} & 12 \\ & (\mathrm{Dec}) \\ & \hline \end{aligned}$ |
| Avg Temp |  |  |  |  |  |  |  |  |  |  |  |  |

Cosine equation (don't use regression):

## Europe City:

$\qquad$ Color on Desmos: $\qquad$

| Month | 1 <br> (Jan) | 2 <br> (Feb) | 3 <br> (Mar) | 4 <br> (April) | 5 <br> (May) | 6 <br> (June) | 7 <br> (July) | 8 <br> (Aug) | 9 <br> (Sept) | 10 <br> (Oct) | 11 <br> (Nov) | 12 <br> (Dec) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Avg <br> Temp |  |  |  |  |  |  |  |  |  |  |  |  |

Cosine equation (don't use regression):
$\qquad$

| Month | 1 <br> (Jan) | 2 <br> (Feb) | 3 <br> (Mar) | 4 <br> (April) | 5 <br> (May) | 6 <br> (June) | 7 <br> (July) | 8 <br> (Aug) | 9 <br> (Sept) | 10 <br> (Oct) | 11 <br> (Nov) | 12 <br> (Dec) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Avg <br> Temp |  |  |  |  |  |  |  |  |  |  |  |  |

Sine equation (use regression on your calculator): $\qquad$

Africa
City: $\qquad$ Color on Desmos: $\qquad$

| Month | 1 <br> (Jan) | 2 <br> (Feb) | 3 <br> (Mar) | 4 <br> (April) | 5 <br> (May) | 6 <br> (June) | 7 <br> (July) | 8 <br> (Aug) | 9 <br> (Sept) | 10 <br> (Oct) | 11 <br> (Nov) | 12 <br> (Dec) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Avg <br> Temp |  |  |  |  |  |  |  |  |  |  |  |  |

Sine equation (use regression on your calculator): $\qquad$

## Australia/Oceania

City: $\qquad$ Color on Desmos: $\qquad$

| Month | 1 <br> (Jan) | 2 <br> (Feb) | 3 <br> (Mar) | 4 <br> (April) | 5 <br> (May) | 6 <br> (June) | 7 <br> (July) | 8 <br> (Aug) | 9 <br> (Sept) $)$ | 10 <br> (Oct) | 11 <br> (Nov) | 12 <br> (Dec) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Avg <br> Temp |  |  |  |  |  |  |  |  |  |  |  |  |

Sine equation (use regression on your calculator):

## Questions:

1. What are some reasons for the differences in temperature graphs between each city?
2. What is the actual meaning for the c value in each equation?
3. Which value(s) (a, b, c, and/or d) would change if January was month 0 ?
