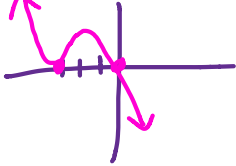Name:_____

Spring Semester Exam Review

All questions should be done without a calculator except for #28-31. A completed review will be worth a curve on your exam.
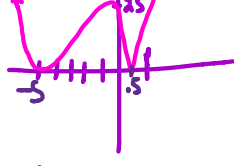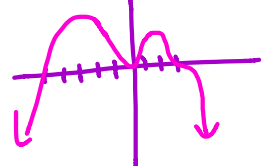
Unit 7 Polynomials

Graph **EB - odd**

1. $y = -x(x+3)^2$

2. $y = (2x-1)^2(x+5)^2$ **+ even**

3. $y = -x^3(x-3)^2(x+5)$ **-odd**

4. Find the remainder when the polynomial $2x^4 + x^3 - x + 2$ is divided by the binomial $x - 2$
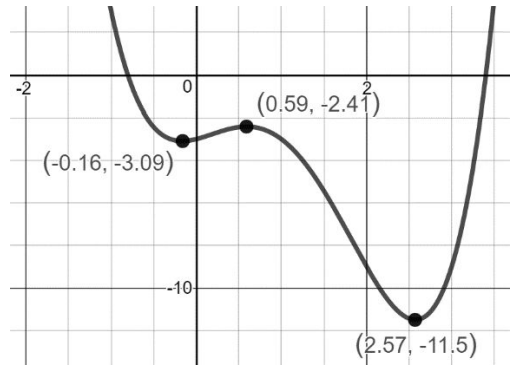
$2(2)^4 + 2^3 - 2 + 2 = 40$ **or**

$$2 \overline{)\begin{array}{ccccc} 2 & 1 & 0 & -1 & 2 \\ & 4 & 10 & 20 & 38 \\ \hline 2 & 5 & 10 & 19 & \boxed{40} \end{array}}$$

5. Find the relative extrema of the function $y = -x(x+3)^2$ and list the intervals where the function is increasing and decreasing

rel min -3.09 @ x= -.16
      -11.5 @ x= 2.57
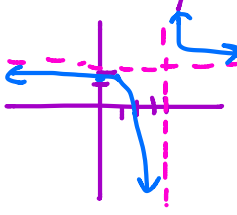rel max -2.41 @ x= .59
inc [-.16, .59] U [2.57, ∞)
dec (-∞, -.16] U [.59, 2.57]

(0.59, -2.41)
(-0.16, -3.09)
(2.57, -11.5)

Unit 8 Rational Functions

Find all asymptotes and graph

6. $\dfrac{2x-3}{x-4}$

VA x=4 HA y=2

7. $\dfrac{x^2-1}{x^2+11x-12}$ $\dfrac{(x+1)(x-1)}{(x+12)(x-1)}$

VA x=-12 RO (1, 2/13)
HA y=1

8. $\dfrac{x+6}{x^2-9}$ $\dfrac{x+6}{(x+3)(x-3)}$

HA y=0 VA x=3, x=-3
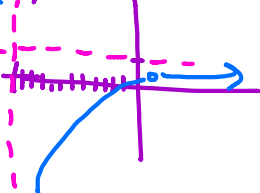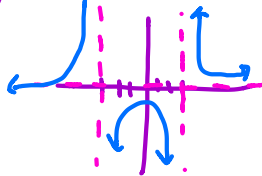
9. $\dfrac{x^2-2x-15}{x+3}$ $\dfrac{(x-5)(x+3)}{x+3}$

hole @ (-3,-8)
no HA, VA

10. $\dfrac{x^2+1}{x+3}$

VA x:-3
noHA

$$-3 \overline{)\begin{array}{ccc} 1 & 0 & 1 \\ & -3 & 9 \\ \hline 1 & -3 & 10 \end{array}}$$

SA y= x-3
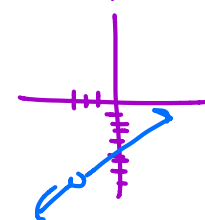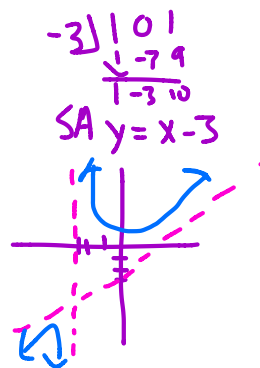
Find the domain of the function

11. $\dfrac{2}{x^2-4x+3}$

$(x-3)(x-1)$
x≠3 x≠1
or
$(-∞,1) U (1,3) U (3,∞)$

12. $\dfrac{2}{3x^2-7x+6}$

$(3x-1)(x-2)$
x≠ 1/3, 2
$(-∞, \frac{1}{3}) U (\frac{1}{3}, 2) U (2, ∞)$

Unit 9 Logs and Exponents

Write as a single log

13. $\log 25 + \log 4$

$\log 100 = 2$

14. $6(\log a + \log b)$

$\log a^6 b^6$ • or $\log(ab)^6$

15. $\ln 2x + 3(\ln x - \ln y)$

$\ln \dfrac{2x \cdot x^3}{y^3} = \ln \dfrac{2x^4}{y^3}$

Solve. Leave answers in terms of ln if needed

16. $5^x = \dfrac{1}{25}$

-2

17. $3^x = 8$

$\log_3 8$ or $\dfrac{\ln 8}{\ln 3}$

18. $27^{2x-1} = 9^{x+3}$

$3^{3(2x-1)} = 3^{2(x+3)}$

$3(2x-1) = 2(x+3)$

$6x - 3 = 2x + 6$

$4x = 9$

$x = 9/4$

19. $\log_4(3x-2) = 2$

$4^2 = 3x - 2$

$18 = 3x$

$x = 18/3$

20. $\log_8 x = \dfrac{2}{3}$

$8^{2/3} = x$

$(\sqrt[3]{8})^2 = \boxed{4}$

21. $5\log_2(\log_3 81)$

$5 \cdot \log_2 4$

$\dfrac{5 \cdot \log_2 4}{2}$

$5 \cdot 2 = \boxed{10}$

22. $5e^x - 12 = 7$

$5e^x = 19$

$e^x = \dfrac{19}{5}$

$\boxed{\ln \dfrac{19}{5} = x}$

23. $\ln 1$

$e^0 = 1$

$\boxed{0}$

24. $\ln e^3$

3

25. $2 = \log_3(9n+10) - \log_3(5n)$

$2 = \log_3 \dfrac{9n+10}{5n}$

$9 = \dfrac{9n+10}{5n}$

$45n = 9n + 10$

$36n = 10$

$n = 3.6$

26. $\ln 5 + \ln(x+2) = \ln 7$

$\ln(5x+10) = \ln 7$

$5x + 10 = 7$

$5x = -3$

$\boxed{x = -3/5}$

27. $\log x + \log(x-9) = 1$

$\log(x^2 - 9x) = 1$

$10^1 = x^2 - 9x$

$0 = x^2 - 9x - 10$

$(x-10)(x+1)$

$x = 10$

You may use a calculator for #28-31. Questions like these will be on the calculator portion of your test.

28. A radioactive substance has a half-life of 420 years. How much remains of a 2 oz. sample after 200 years?

$1 = 2e^{420r}$ $r = -.00165$

$.5 = e^{420r}$

$\ln .5 = 420r$

$x = 2e^{200(-.00165)}$

29. A bacteria culture has an initial count estimate of 4000. After 20 minutes the count is 22,400. What is the growth rate, and approximately how many minutes did it take for the culture to double?

$22400 = 4000e^{20r}$ $r = .08614$

$5.6 = e^{20r}$

$\ln 5.6 = 20r$

$8000 = 4000e^{.08614t}$

$2 = e^{.08614t}$

$\dfrac{\ln 2}{.08614} = \dfrac{.08614t}{.08614}$

30. $3000 is invested in an account that pays 5% annual interest. How much more money would you make if the interest was compounded daily for 10 years instead of compounded quarterly?

$3000\left(1 + \dfrac{.05}{365}\right)^{10(365)}$

$3000\left(1 + \dfrac{.05}{4}\right)^{10 \cdot 4}$

31. How many years will it take an investment of $1000 to double itself when interest is compounded continuously at 6% annually?

$2000 = 1000e^{.06t}$

$2 = e^{.06t}$

$\dfrac{\ln 2}{.06} = \dfrac{.06t}{.06}$

$\boxed{11.55 = t}$

Unit 10 Conics

Tell which conic is represented by the equations

32. $2x + 3y^2 + 17 = 0$ parabola

33. $8x^2 + 4y^2 + 3x - 2y + 17 = 0$

ellipse

34. $-3x^2 - 3y^2 + 4x - 2y + 5 = 0$

circle

35. $6x^2 - 2y^2 + 7x + 2y - 4 = 0$

hyperbola

Write the equation in standard form and draw a graph.

36. $\dfrac{-4(x-1)^2}{36} + \dfrac{9(y-3)^2}{36} = \dfrac{36}{36}$

$-\dfrac{(x-1)^2}{9} + \dfrac{(y-3)^2}{4} = 1$

37. An ellipse with endpoints

center $(-1, 2)$

$\dfrac{(x+1)^2}{9} + \dfrac{(y-2)^2}{25} = 1$

38. $x^2 + y^2 + 6x + 4y + 12 = 0$

$x^2 + 6x + 9 + y^2 + 4y + 4 = -12 + 4 + 9$

$(x+3)^2 + (y+2)^2 = 1$

$C(-3, -2)$

$r = 1$

Unit 11 Series and Sequences

39. Write the first 5 terms of the sequence whose nth term is $a_n = n^2 - 3$

$a_1 = -2 \quad a_2 = 1 \quad a_3 = 6 \quad a_4 = 13 \quad a_5 = 22$

40. If $a_n = a_{n-1} + 5$ and $a_1 = 3$, find a_2, a_3 and a_4

$a_2 = 3 + 5 = 8 \qquad a_4 = 13 + 5 = 18$
$a_3 = 8 + 5 = 13$

41. Evaluate $\displaystyle\sum_{x=5}^{9} |4 - x|$

$|4-5| + |4-6| + |4-7| + |4-8| + |4-9|$
$1 + 2 + 3 + 4 + 5 = \boxed{15}$

42. Evaluate $\displaystyle\sum_{c=0}^{4} (-2)^c$

$(-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4$
$1 + -2 + 4 + -8 + 16$
$\boxed{11}$

43-44: Rewrite using sigma notation:

43. $-4 - 8 - 12 - 16 - 20$

$\displaystyle\sum_{n=1}^{5} -4n \quad \text{or} \quad \sum_{n=1}^{5} -4 - 4(n-1)$

44. $12 + 3 - 6 - 15$

$\displaystyle\sum_{n=1}^{4} 12 - 9(n-1)$

Determine whether the sequence is arithmetic, geometric or neither. If arithmetic or geometric, find the nth term.

45. 5, 7, 9, 11, 13...

Arithmetic
$5 + 2(n-1)$

46. 1/7, 1, 7, 49

Geometric
$\frac{1}{7}(7)^{n-1}$

47. 15, 17, 20, 23, 25...

neither

Find the sum of the series

48. $\dfrac{1}{3} - \dfrac{2}{3} + \dfrac{4}{3} - \dfrac{8}{3} \ldots$

diverges
$r = -2$

49. $25 + 5 + 1 + \ldots$

$\dfrac{25}{1 - \frac{1}{5}} = 25 \div \frac{4}{5} = 25 \cdot \frac{5}{4} = \frac{125}{4}$ or
31.25

Unit 12 Polar and Parametric
Convert from polar coordinates to rectangular coordinates

50. $\left(5, \dfrac{\pi}{6}\right)$

$x = 5\cos\frac{\pi}{6} = 5\left(\frac{\sqrt{3}}{2}\right)$
$y = 5\sin\frac{\pi}{6} = 5\left(\frac{1}{2}\right)$
$\left(\dfrac{5\sqrt{3}}{2}, \dfrac{5}{2}\right)$
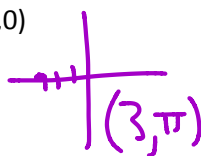
51. $\left(2, \dfrac{3\pi}{4}\right)$

$x = 2\cos\frac{3\pi}{4} = 2\left(\frac{-1}{\sqrt{2}}\right)$
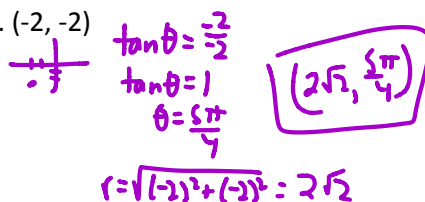$y = 2\sin\frac{3\pi}{4} = 2\left(\frac{1}{\sqrt{2}}\right)$
$\left(\dfrac{-2}{\sqrt{2}}, \dfrac{2}{\sqrt{2}}\right)$

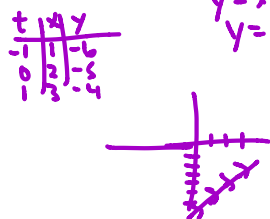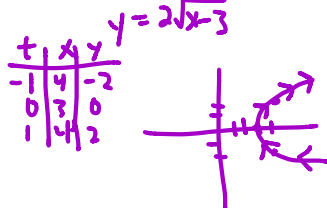Convert from rectangular coordinates to polar coordinates

52. (-3,0)

$(3, \pi)$

53. (-2, -2)

$\tan\theta = \frac{-2}{-2}$

$\tan\theta = 1$

$\theta = \frac{5\pi}{4}$

$\left(2\sqrt{2}, \frac{5\pi}{4}\right)$

$r = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$

Eliminate the parameter and draw a graph for the following parametric equations

54. x = t + 2 $x - 2 = t$
    y = t − 5 $y = x - 2 - 5$
               $y = x - 7$

t	x	y
-1	1	-6
0	2	-5
3	3	-4

55. x = 3 + t² $x - 3 = t^2$
    y = 2t $\pm\sqrt{x-3} = t$
             $y = 2\sqrt{x-3}$

t	x	y
-1	4	-2
0	3	0
1	4	2

56. x = 2cost $x^2 + y^2 = 4$
    y = 2sint

t	x	y
0	2	0
π/2	0	2
π	-2	0

57. x = 3cost $\frac{x^2}{9} + \frac{y^2}{4} = 1$
    y = 2sin(-t)

t	x	y
0	3	0
π/2	0	-2
π	-3	0
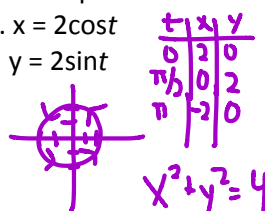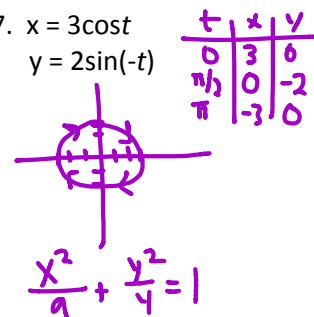
Use the graph of f(x) below to find the following limits
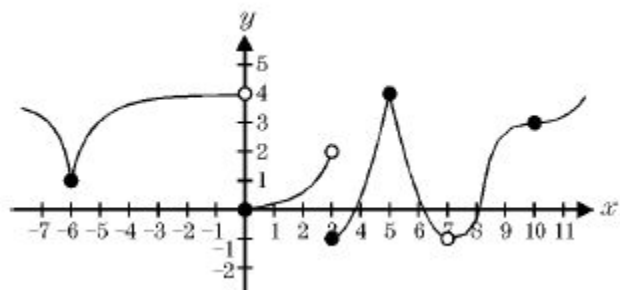
58. $\lim_{x \to -6} f(x) = 1$

59. $\lim_{x \to 0} f(x) = DNE$

60. $\lim_{x \to 3} f(x) = DNE$

61. $\lim_{x \to 5} f(x) = 4$

62. $\lim_{x \to 7} f(x) = -1$

63. $\lim_{x \to 10} f(x) = 3$

64. $\lim_{x \to 0^-} f(x) = 4$

65. $\lim_{x \to 0^+} f(x) \neq 0$

66. $\lim_{x \to 3^-} f(x) = 2$

67.

$\lim_{x \to 3^+} f(x) = -1$