Name: _____

Unit 10 Log Review

Remember to study your notes, homework, and quiz. The "Ultimate Log Worksheet" is also a GREAT review!

1. Convert the following into LOGARITHMIC form.

A. $2^9 = 512$ $\log_2 512 = 9$

B. $9^0 = 1$ $\log_9 1 = 0$

C. $5^{-3} = \frac{1}{125}$ $\log_5 \frac{1}{125} = -3$

D. $8^{\frac{1}{3}} = 2$ $\log_8 2 = \frac{1}{3}$

2. Convert the following into EXPONENTIAL form.

A. $\log_6 1296 = 4$ $6^4 = 1296$

B. $\log_7 7 = 1$ $7^1 = 7$

C. $\log_6 \frac{1}{36} = -2$ $6^{-2} = \frac{1}{36}$

D. $\log_{16} 4 = \frac{1}{2}$ $16^{1/2} = 4$

Solve each equation for x:

3. $\log_3 81 = x$ $\boxed{x=4}$

$3^x = 81 \Rightarrow 3^x = 3^4$

4. $\log_{\sqrt{2}} x = -3$

$\sqrt{2}^{-3} = x \Rightarrow (2^{\frac{1}{2}})^{-3} = \boxed{\frac{1}{2^{3/2}}}$

5. $\log_4 x = \frac{1}{2}$

$4^{\frac{1}{2}} = x$

$\boxed{2 = x}$

6. $\log_x \frac{1}{32} = -5$ $\boxed{x=2}$

$x^{-5} = \frac{1}{32} \Rightarrow \frac{1}{x^5} = \frac{1}{32}$

7. $\log_5 (x-4) = 0$

$5^0 = x-4$

$1 = x-4 \Rightarrow \boxed{x=5}$

$+4 \quad +4$

8. $\log_b b^4 = x$

$b^x = b^4$

$\boxed{x=4}$

9. Use the laws of logarithms to expand the expression

A. $\log_2 (xy)^{10}$

$10\log_2(xy) = \boxed{10\log_2 x + \log_2 y}$

B. $\log_a \left(\frac{x^2}{yz^3}\right)$

$2\log_a x - \log_a y - 3\log_a z$

10. Use the Laws of Logarithms to combine the expression into a single log

A. $\log_5 (x^2-1) - \log_5 (x-1)$

$\log_5\left(\frac{x^2-1}{x-1}\right) = \log_5\left(\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\right) = \boxed{\log_5(x+1)}$

B. $\ln(a+b) + \ln(a-b) - 2\ln c$

$\ln\left(\frac{a^2-b^2}{c^2}\right)$

Solve for *x*: *leave answers as simplified fractions as needed*

11. $3^{2x-1} = 27$

$3^{2x-1} = 3^3$ $\boxed{x=2}$

$2x-1 = 3$

$2x = 4$

12. $3^x = \left(\frac{1}{3}\right)^{x-3}$ $\boxed{x = 3/2}$

$3^x = 3^{-1(x-3)}$

$x = -x+3 \Rightarrow 2x = 3$

13. $\sqrt{9}^{5x-1} = \left(\frac{1}{81}\right)^{x-4}$

$9^{\frac{1}{2}(5x-1)} = 9^{-2(x-4)}$

$\frac{1}{2}(5x-1) = -2(x-4)$ $\boxed{x = 17/9}$

14. $\log x = \frac{1}{3}\log 64 - \frac{1}{5}\log 32$

$\log x = \log \frac{\sqrt[3]{64}}{\sqrt[5]{32}}$

$x = \frac{4}{2} = \boxed{2}$

15. $\log_4 (x-4) + \log_4 (x+4) = 2\log_4 3$

$\log_4((x-4)(x+4)) = \log_4 3^2$

$x^2 - 16 = 9$

$x^2 = 25$

$\boxed{x=5}$

16. $\log_2 x + \log_2 (x-6) = 4$

$\log_2(x(x-6)) = 4$

$2^4 = x^2 - 6x$

$0 = x^2 - 6x - 16$

$(x-8)(x+2)$ $\boxed{8}$

$\{\cancel{-2}, 8\}$

17. Evaluate:

A. $49^{-\log_7 4}$

$7^{2(-\log_7 4)}$

$7^{-2\log_7 4}$

$7^{\cancel{\log_7} 4^{-2}}$

$\boxed{\frac{1}{16}}$

B. $6^{3\log_6 2 + 2\log_6 3}$

$\cancel{6^{\log_6}} 2^3 \cdot 3^2$

$8 \cdot 9$

$\boxed{72}$

C. $\log_2 (\log_2 (\log_2 16))$

$\log_2 (\log_2 (4))$

$\log_2 (2)$

$\boxed{1}$

D. $\log_{16} (\log_2 (\log_3 9))$

$\log_{16} (\log_2 2)$

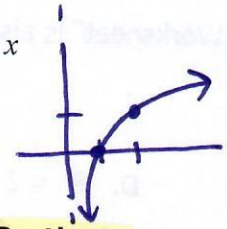
$\log_{16} (1)$

$\boxed{0}$

Graph each function

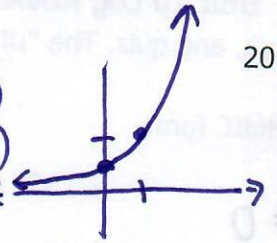
18. $y = \log_2 x$

(1, 0)
(2, 1)
x = 0



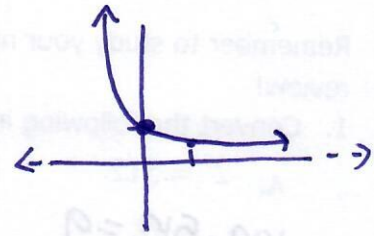
19. $y = 2^x$

(0, 1)
(1, 2)
y = 0



20. $y = \left(\frac{1}{2}\right)^x$

(0, 1)
(1, ½)
y = 0



Calculator Portion

Solve for *x*: *Round to 2 decimal places*

21. $\ln x = -4.2$

$e^{-4.2} = x$

$\boxed{x \approx 0.01}$

22. $2.13^x = 6.3$

$\log_{2.13} 6.3 = x$

$\boxed{x \approx 2.43}$

23. $\frac{3e^{5x}}{3} = \frac{12}{3}$

$e^{5x} = 4$

$\frac{\ln 4}{5} = \frac{5x}{5}$

$\boxed{x \approx 0.28}$

24. $\frac{3 \ln 5x}{3} = \frac{27}{3}$

$\ln 5x = 9$

$\frac{e^9}{5} = \frac{5x}{5}$

$\boxed{x \approx 1620.62}$

25. $3e^{-x} - 4 = 9$

$+4 \quad +4$

$\frac{3e^{-x}}{3} = \frac{13}{3}$

$e^{-x} = 13/3$

$\frac{\ln\left(\frac{13}{3}\right)}{-1} = \frac{-x}{-1}$

$\boxed{x \approx -1.47}$

26. $\ln(2x+7) = -3$

$e^{-3} = 2x + 7$

$\frac{e^{-3} - 7}{2} = x$

$\boxed{x \approx -3.48}$

Formulas: $y_t = y_0 e^{rt}$ $\quad A = Pe^{rt}$ $\quad A = P\left(1 + \frac{r}{n}\right)^{nt}$

27. The half life of a certain substance is 18 days. If there are 8.3 grams initially, when will there be 0.5 grams left? *Round to 2 decimal places*

① Find r

A = 8.3/2 = 4.15
P = 8.3
t = 18

$4.15 = 8.3e^{r(18)}$

r = -.0385

② Find new t

A = 0.5
P = 8.3
t = ?

$0.5 = 8.3e^{(-.0385)t}$

$\frac{\ln(.060\ldots)}{-.0385} = t$

$\boxed{72.96 \text{ days}}$

28. What is the total value after 7 years of an initial investment of $5250 that earns interest at the rate of 6.1%, compounded monthly?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

P = 5250 n = 12
r = 0.061 t = 7

$A = 5250\left(1 + \frac{.061}{12}\right)^{12 \cdot 7}$

$\boxed{\$8037.73}$

29. A $2500 investment earns interest compounded quarterly. Determine the interest rate needed in order for the money to grow to $4000 over the course of 4 years. *Give the answer as a percentage rounded to 1 decimal.*

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

A = 4000 P = 2500 n = 4
r = ? t = 4

$4000 = 2500\left(1 + \frac{r}{4}\right)^{4 \cdot 4}$

$1.6 = \left(1 + \frac{r}{4}\right)^{16}$

$1.0298\ldots = 1 + \frac{r}{4}$

r = 0.1192

$\boxed{11.9\%}$

30. How long will it take an investment of $1100 at 7.45% APR to grow to $2500 if the interest rate is compounded monthly? *Round to 1 decimal place*

A = 2500 r = .0745
P = 1100 n = 12

$2500 = 1100\left(1 + \frac{.0745}{12}\right)^{12t}$

$2.27\ldots = (1.006\ldots)^{12t}$

$\frac{\log_{1.006} 2.27}{12} = \frac{12t}{12}$

$\boxed{t = 11.05 \text{ years}}$

31. How long will it take an investment of $3000 to double if it is invested in an account earning 4.75% interest compounded continuously?

$A = Pe^{rt}$

A = 6000 r = .0475
P = 3000 t = ?

$6000 = 3000e^{.0475t}$

$2 = e^{.0475t}$

$\frac{\ln 2}{.0475} = t$

$\boxed{t = 14.59 \text{ years}}$