

Name: _____

Polar and Parametric Test Review

Part 1: Polar (all non-calculator)

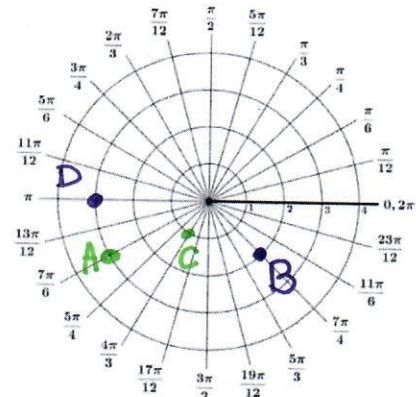
Plot the coordinates and convert from polar coordinates to rectangular coordinates

$$1. A\left(3, \frac{7\pi}{6}\right) \quad x = 3 \cos \frac{7\pi}{6} = 3\left(-\frac{\sqrt{3}}{2}\right) \quad y = 3 \sin \frac{7\pi}{6} = 3\left(-\frac{1}{2}\right) \quad \boxed{\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)}$$

$$2. B\left(-2, \frac{3\pi}{4}\right) \quad x = -2 \cos \frac{3\pi}{4} = -2\left(-\frac{\sqrt{2}}{2}\right) \quad y = -2 \sin \frac{3\pi}{4} = -2\left(\frac{\sqrt{2}}{2}\right) \quad \boxed{(\sqrt{2}, -\sqrt{2})}$$

$$3. C\left(1, -\frac{2\pi}{3}\right) \quad x = 1 \cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2} \quad y = 1 \sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2} \quad \boxed{\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$$

$$4. D(-3, 0) \quad x = -3 \cos 0 = -3(1) \quad y = -3 \sin 0 = -3(0) \quad \boxed{(-3, 0)}$$



Convert from rectangular coordinate to polar coordinates many correct answers

$$5. (2, -2) \quad x^2 + y^2 = r^2 \quad r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2} \quad \tan \theta = -\frac{2}{2} \quad \theta = -\frac{\pi}{4} \quad \boxed{(2\sqrt{2}, -\frac{\pi}{4})}$$

$$6. (3, 3\sqrt{3}) \quad r = \sqrt{3^2 + (3\sqrt{3})^2} = \sqrt{18+27} = \sqrt{45} = 3\sqrt{5} \quad \theta = \tan^{-1}\left(\frac{3\sqrt{3}}{3}\right) = 60^\circ \quad \boxed{(3\sqrt{5}, 60^\circ)}$$

$$7. (0, -4) \quad \theta = \frac{3\pi}{2} \quad \boxed{(4, \frac{3\pi}{2})}$$

no formulas necessary

Convert the equation from rectangular to polar

$$8. x^2 + y^2 = 25 \quad \text{circle w/ radius 5} \quad \frac{x^2 + y^2}{r^2} = 25 \quad r^2 = 25 \quad \boxed{r = 5}$$

$$9. y = 2 \quad \text{horizontal line}$$

$$y = rs \sin \theta \quad \frac{rs \sin \theta}{\sin \theta} = 2 \quad \frac{r \sin \theta}{\sin \theta} = 2 \quad r = 2 \csc \theta$$

Convert the equation from polar to rectangular

$$10. r \cos \theta = 4 \quad x = 4 \quad \boxed{x=4}$$

$$11. r = 4 \sin \theta \quad \text{circle not centred @ origin} \rightarrow \text{graph it! center } (0, 2) \text{ radius 2}$$

$$\boxed{x^2 + (y-2)^2 = 4}$$

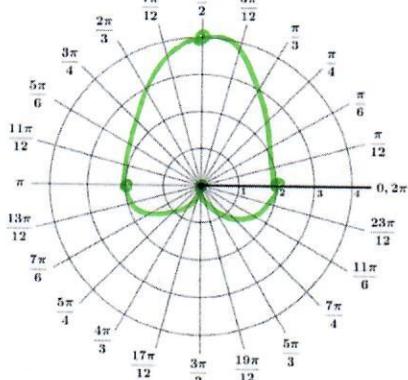
Write the type of graph, make a table, and draw the graph

$$12. r = 2 + 2 \sin \theta$$

Type: **cardioid**

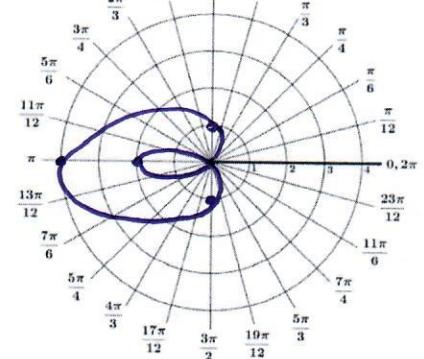
r	θ
$2 + 2(0) = 2$	0
$2 + 2(1) = 4$	$\frac{\pi}{2}$
$2 + 2(0) = 2$	π
$2 + 2(-1) = 0$	$3\pi/2$

010 chart



$$13. r = 1 - 3 \cos \theta \quad \text{Type: limagon w/ loop}$$

r	θ
$1 - 3(1) = -2$	0
$1 - 3(0) = 1$	$\frac{\pi}{2}$
$1 - 3(-1) = 4$	π
$1 - 3(0) = 1$	$3\pi/2$

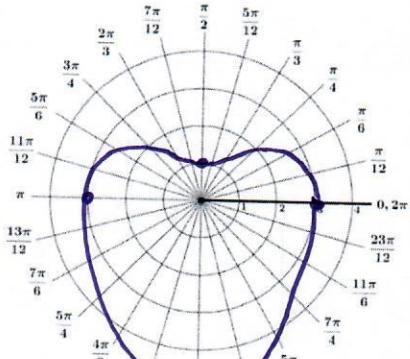


Write the type of graph, make a table, and draw the graph

$$14. r = 3 - 2 \sin \theta$$

Type: dimpled limacon

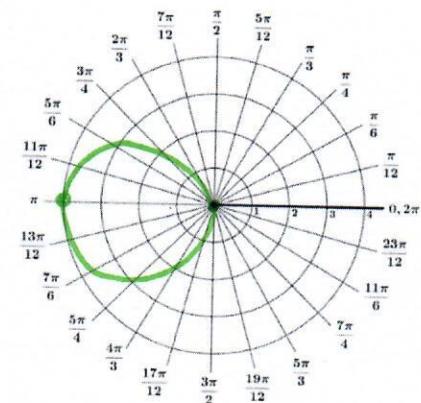
r	θ
$3 - 2(0) = 3$	0
$3 - 2(1) = 1$	$\frac{\pi}{2}$
$3 - 2(0) = 3$	π
$3 - 2(-1) = 5$	$\frac{3\pi}{2}$



$$15. r = -4 \cos \theta$$

Type: circle

r	θ
-4(1) = -4	0
-4(0) = 0	$\frac{\pi}{2}$
-4(-1) = 4	π
-4(0) = 0	$\frac{3\pi}{2}$



15. Describe the difference between the graph of A) $r = 2 \sin 3\theta$ and B) $r = 3 \sin 2\theta$

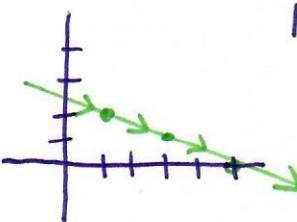
roses, but rose A has 3 petals of length 2 and rose B has 4 petals of length 3
(2n since n is even)

Part 2: Parametric (non-calculator)

Graph the parametric equations (show direction with arrows) and eliminate the parameter to find a rectangular equation

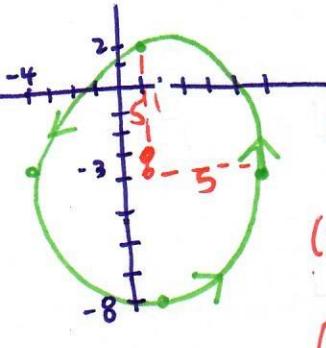
$$1. \begin{aligned} x &= 2t + 3 \\ y &= -t + 1 \\ t &= -y+1 \end{aligned}$$

t	x	y
-1	1	2
0	3	1
1	5	0



$$4. \begin{aligned} x &= 5 \cos(t) + 1 \\ y &= 5 \sin(t) - 3 \end{aligned}$$

t	x	y
0	6	-3
$\frac{\pi}{2}$	1	2
π	-4	-3
$\frac{3\pi}{2}$	1	-8

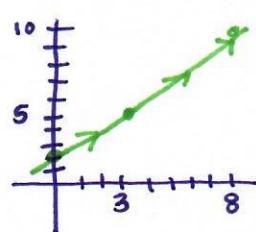


circle centered at (1, -3)

$$(x+3)^2 + (y-1)^2 = 25$$

$$2. \begin{aligned} x &= t^2 - 1 \\ y &= t^2 + 1 \end{aligned}$$

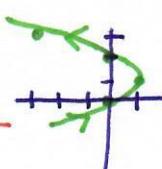
$$1 \leq t \leq 3$$



t	x	y
1	0	2
2	3	5
3	8	10

$$\begin{aligned} t &= \sqrt{x+1} \\ y &= (\sqrt{x+1})^2 + 1 \\ y &= x + 2 \end{aligned}$$

$$3. \begin{aligned} x &= 1 - t^2 \\ y &= 1 + t \end{aligned}$$



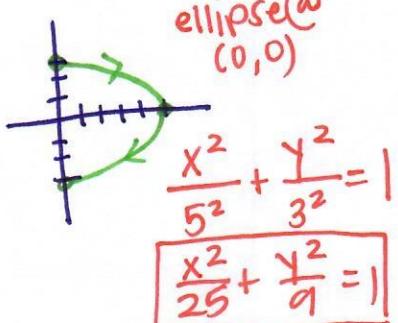
t	x	y
-1	0	0
0	1	1
1	0	2
2	-3	3

$$\begin{aligned} t &= \pm \sqrt{-x+1} \\ y &= 1 \pm \sqrt{-x+1} \end{aligned}$$

$$5. \begin{aligned} x &= 5 \sin(t) \\ y &= 3 \cos(t) \end{aligned}$$

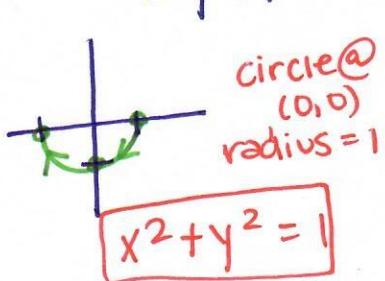
$$0 \leq t \leq \pi$$

t	x	y
0	0	3
$\frac{\pi}{2}$	5	0
π	0	-3



ellipse @ (0, 0)

$$\begin{aligned} \frac{x^2}{5^2} + \frac{y^2}{3^2} &= 1 \\ \frac{x^2}{25} + \frac{y^2}{9} &= 1 \end{aligned}$$

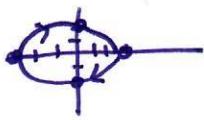


circle @ (0, 0)
radius = 1

$$x^2 + y^2 = 1$$

Write a set of parametric equations to model the following situations

7. An ellipse, centered at the origin, with end points $(0, -2)$, $(0, 2)$, $(-3, 0)$, $(3, 0)$ that rotates clockwise

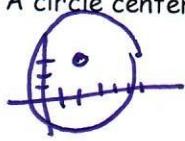


$$x = 3 \sin T$$

$$y = 2 \cos T$$

\sin first

8. A circle centered at $(2, 3)$ with a radius of 4 that rotates counter-clockwise



$$x = 2 + 4 \cos T$$

\cos first

$$y = 3 + 4 \sin T$$

9. A wall 300 feet away that is 20 ft tall with

a) $t\text{-max } 10$ $20/10 = 2$

$$x = 300$$

$$y = 2T$$

b) $t\text{-max } 2$ $20/2 = 10$

$$x = 300$$

$$y = 10T$$

c) $t\text{-max } 5$ $20/5 = 4$

$$x = 300$$

$$y = 4T$$

10. A cannon shoots a ball from 5 feet off the ground at an initial velocity of 200 ft/sec at an angle of 39° , with a breeze blowing with the ball at 5 ft/sec

$$x = 200 \cos(39^\circ)T + 5T$$

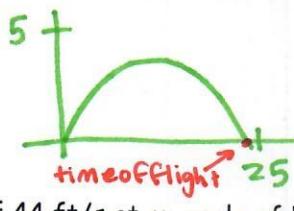
$$y = 200 \sin(39^\circ)T - 16T^2 + 5$$

Part 3 Parametric-Calculator

11. A long jumper leaves the ground with an initial velocity of 31 ft/sec at an angle of 22-degrees. Determine the time of flight, the horizontal distance traveled, and the peak height of the long-jumper.

$$x = 31 \cos(22^\circ)T$$

$$y = 31 \sin(22^\circ)T - 16T^2$$



Time of Flight $y=0$ about 0.72 sec
 $0 = 31 \sin(22^\circ)T - 16T^2$

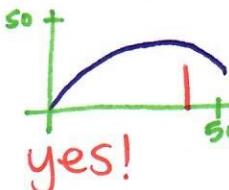
Horizontal distance $x = 31 \cos(22^\circ)(0.72)$ 20.69 ft

Peak Height $T = 0.32/2$ half of flight 2.1 ft

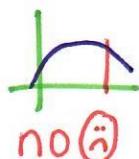
12. A football is kicked with an initial velocity of 44 ft/s at an angle of 55-degrees. If the cross bar of the goal post is 10 feet high and 45 feet away, will the kick be good? What if the wind was blowing in at 3 ft/s?

football $\begin{cases} x = 44 \cos(55^\circ)T \\ y = 44 \sin(55^\circ)T - 16T^2 \end{cases}$

goal post $\begin{cases} x = 45 \\ y = T \quad (\text{Tmax } 10) \end{cases}$



Part 2
 $x = 44 \cos(55^\circ)T - 3T$
 $y = 44 \sin(55^\circ)T - 16T^2$
 same goal post



13. A baseball player hits a ball with an initial velocity of 130 ft/sec. There is a wind blowing with the ball at 3 ft/sec. The angle of elevation of the ball off the bat is 25° and the ball hits the bat 2 ft off the ground. Give the set of parametric equations for the path of the ball. At what time is the ball 200 ft away? Will the ball clear a 10 ft high fence 400 ft away?

$$x = 130 \cos(25^\circ)T + 3T$$

$$y = 130 \sin(25^\circ)T - 16T^2 + 2$$

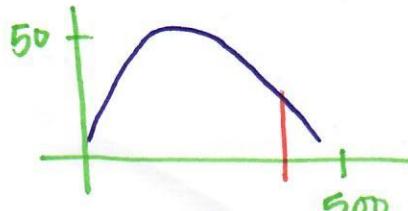
$$x = 200$$

$$200 = 130 \cos(25^\circ)T + 3T$$

$$\frac{200}{120.82} = \frac{120.82}{120.82}T$$

$$T = 1.66 \text{ sec}$$

fence $\begin{cases} x = 400 \\ y = T \quad (\text{Tmax } 10) \end{cases}$



NO $\textcircled{\text{a}}$