Review for Rational Functions

KEY

1. $f(x) = \dfrac{x+5}{x^2+3x-10} = \dfrac{\cancel{x+5}}{\cancel{(x+5)}(x-2)} = \dfrac{1}{x-2}$

low / high Horizontal asymptote: $y=0$

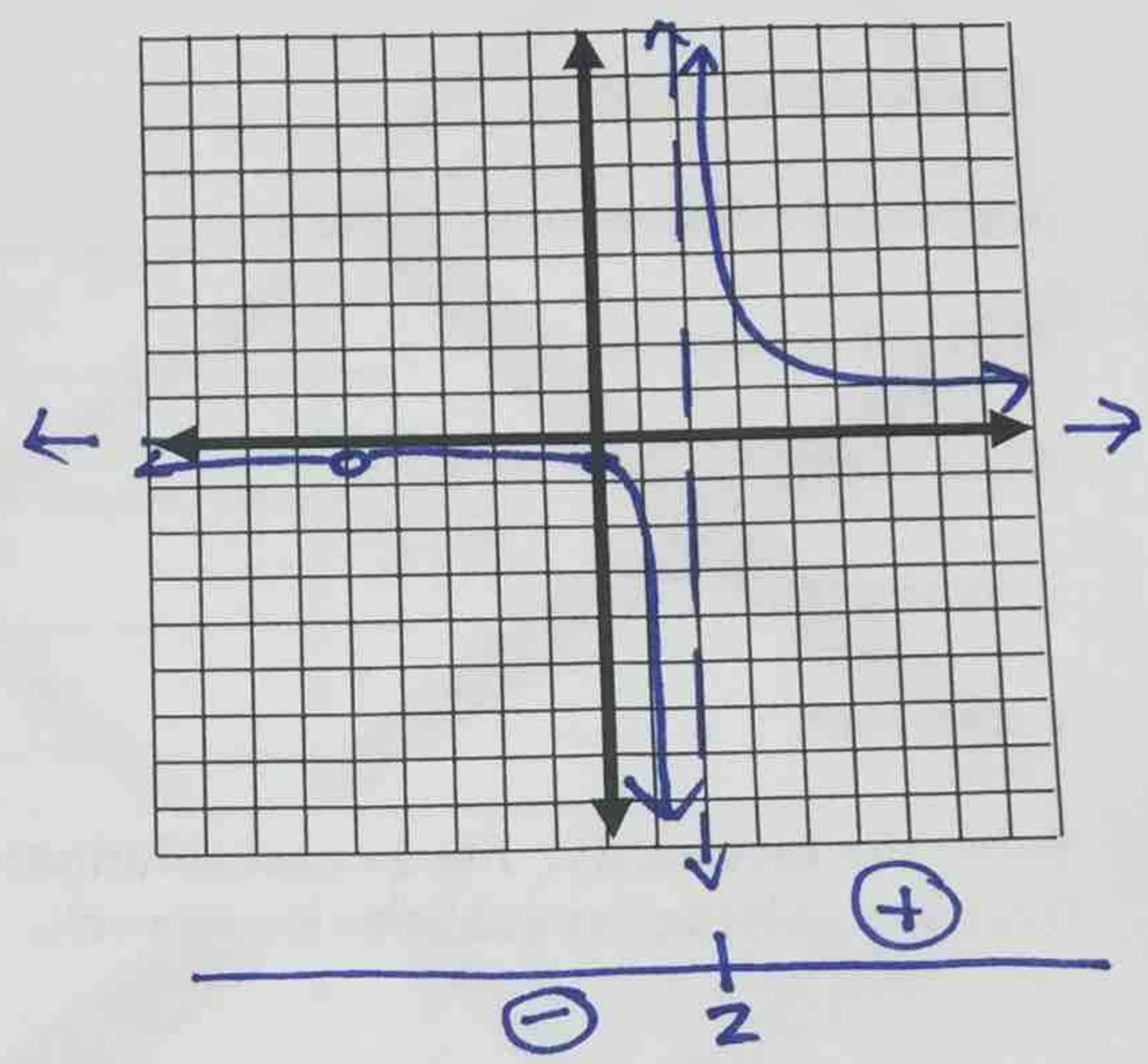
Removable discontinuity (hole): $\left(-5, \frac{1}{-7}\right)$

Vertical asymptote: $x=2$
$x-2=0$

Slant asymptote: none

x-intercept: none
top=0

y-intercept: $\left(0, \frac{-1}{2}\right)$
$x=0$



2. $g(x) = \dfrac{4x^2-1}{x^2-9} = \dfrac{(2x-1)(2x+1)}{(x-3)(x+3)}$

same / same Horizontal asymptote: $y=4$

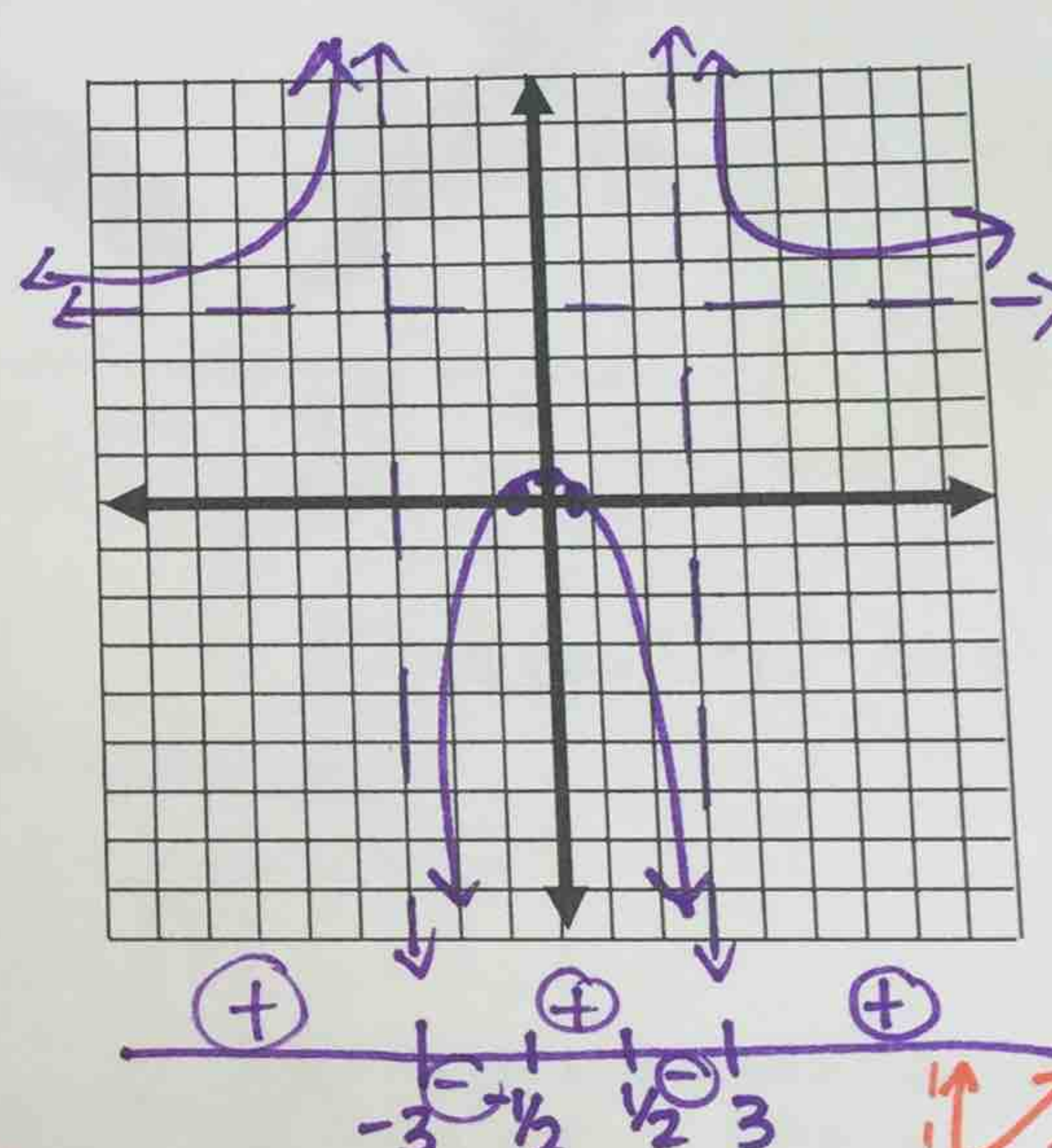
Removable discontinuity (hole): none

Vertical asymptote: $x=3$ and $x=-3$
$x-3=0$ $x+3=0$

Slant asymptote: none

x-intercept: $\left(\frac{1}{2}, 0\right)$ and $\left(\frac{-1}{2}, 0\right)$
$2x-1=0$
$2x+1=0$

y-intercept: $(0, 1/9)$
$x=0$



3. $h(x) = \dfrac{(x^3-25x)}{(x^2-4x-21)} = \dfrac{x(x^2-25)}{(x-7)(x+3)} = \dfrac{x(x-5)(x+5)}{(x-7)(x+3)}$

Horizontal asymptote: none

$$\begin{array}{r} x+4 \\ x^2-4x-21 \overline{)\, x^3-0x^2-25x+0} \\ -(x^3-4x^2-21x) \\ 4x^2-4x \\ -(4x^2-16x-84) \end{array}$$

Removable discontinuity (hole): none

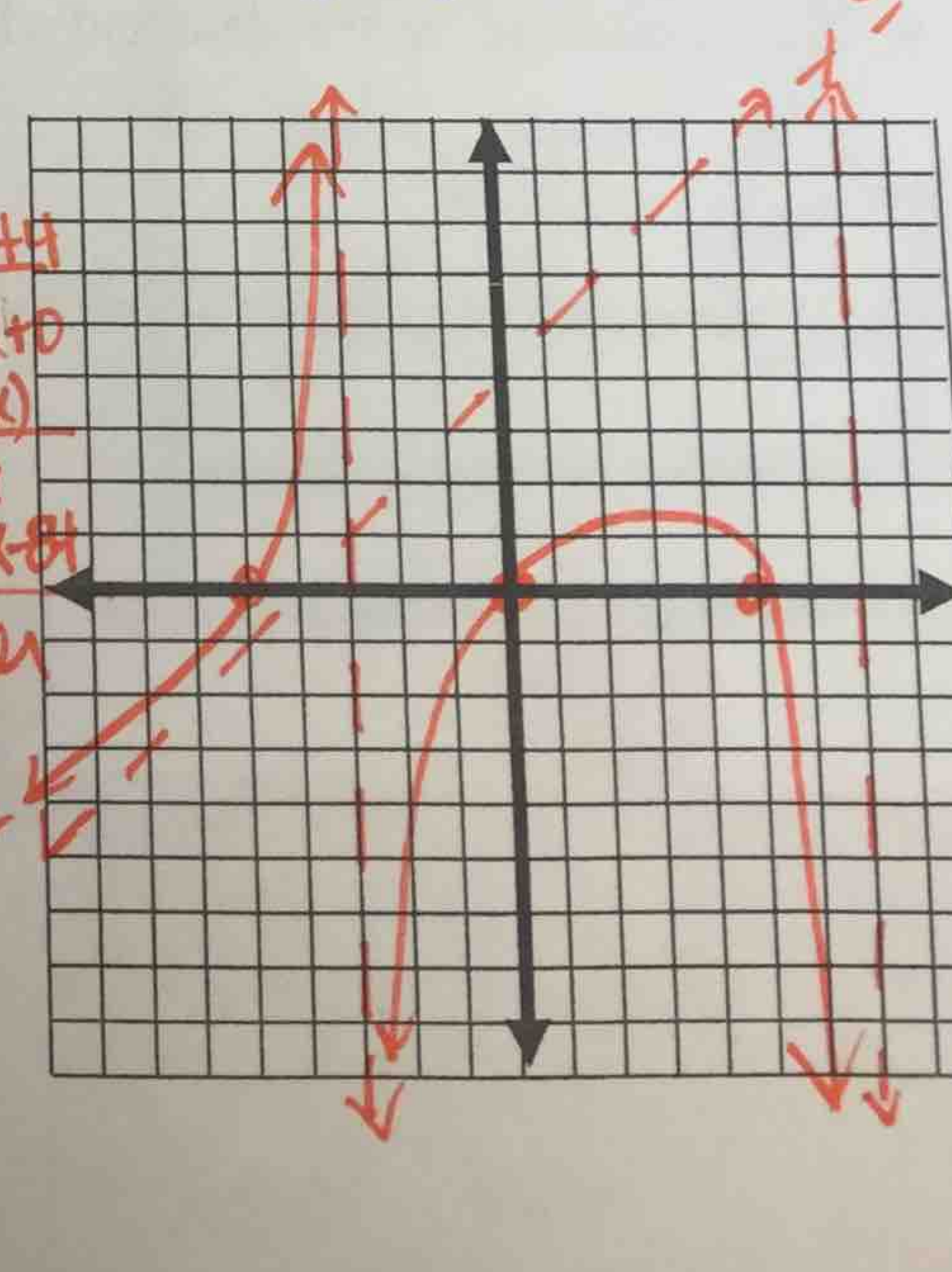
Vertical asymptote: $x=7$ and $x=-3$

Remainder doesn't matter

Slant asymptote: $y=x+4$

x-intercept: $(0,0)$ & $(5,0)$ & $(-5,0)$
$x=0$
$x-5=0$
$x+5=0$

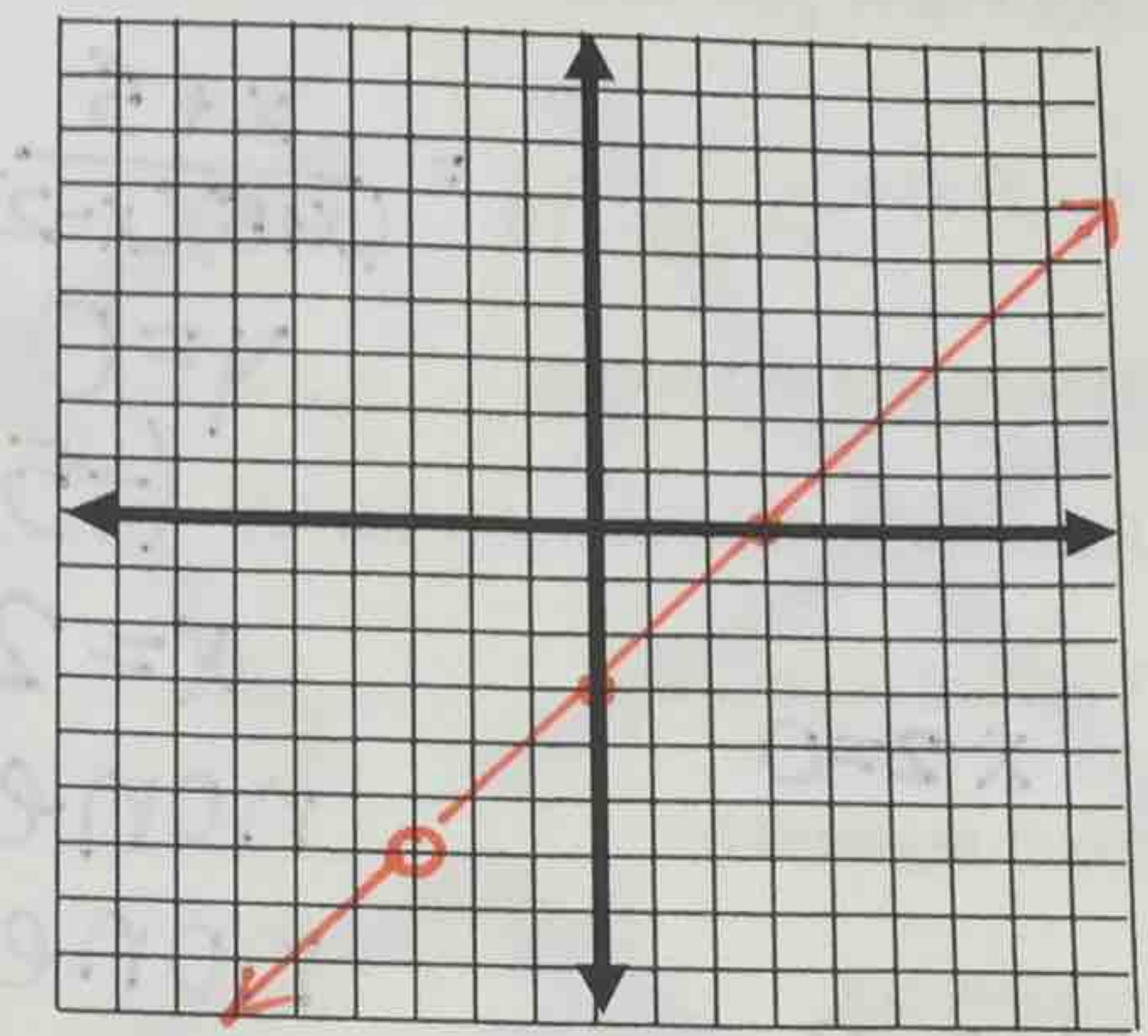
y-intercept: $(0,0)$



+21, 10

4. $h(x) = \frac{(x-3)(x+3)}{(x+3)} = x-3$

line w/ a hole



Horizontal asymptote: None

Removable discontinuity (hole): $(-3,-6)$

Vertical asymptote: None

Slant asymptote: None

x-intercept: $(3,0)$

y-intercept: $(0,-3)$

Solve the inequality. Find exact solutions when possible. If you need more practice, look at the evens from the solving inequalities homework.

5. $\frac{2}{x+3} + 2 \le 3$ ← neg

$\frac{2}{x+3} - 1 \le 3$

$\frac{2}{x+3} - \frac{x+3}{x+3} \le 3$

$\frac{-x-1}{x+3} \le 3$

VA: $x=-3$ ← Asymptote, can't equal!

x-int: $(-1,0)$

$-$ -3 $+$ -1 $-$

$\boxed{(-\infty,-3)\cup[-1,\infty)}$

6. $x^3 - 2x^2 - 3x + 10 < 4x - 4$ ← Neg.

$(x^3 - 2x^2)(-7x + 14) < 0$

$x^2(x-2) - 7(x-2) < 0$

$(x^2-7)(x-2) < 0$

$-$ $-\sqrt{7}$ $+$ 2 $-$ $\sqrt{7}$ $+$

$\boxed{(-\infty,-\sqrt{7})\cup(2,\sqrt{7})}$

7. $4x^4 - 17x^2 + 4 \ge 0$ ← positive

$(4x^4 - 16x^2)(-1x^2 + 4) \ge 0$

$4x^2(x^2-4) - 1(x^2-4) \ge 0$

$(4x^2-1)(x^2-4) \ge 0$

$(2x-1)(2x+1)(x-2)(x+2) \ge 0$

$+$ -2 $-$ $-\frac{1}{2}$ $+$ $\frac{1}{2}$ $-$ 2 $+$

$\boxed{(-\infty,-2]\cup[-\frac{1}{2},\frac{1}{2}]\cup[2,\infty)}$

8. $\frac{x+3}{x^2-2x-8} \ge 0$ ← pos.

$\frac{x+3}{(x-4)(x+2)} \ge 0$

VA: $x=4$ & $x=-2$

x-int: $(-3,0)$

$-$ -3 $+$ -2 $-$ 4 $+$

$\boxed{[-3,-2)\cup(4,\infty)}$

spaghetti sauce

9. Is it possible to have a slant and a horizontal asymptote in the same graph?

NO

10. What is the only type of asymptote that cannot be crossed?

vertical

11. Given the following functions, what are the horizontal asymptotes if the exist?

a) $\frac{1x^2+2}{2x^2-3}$ $y=\frac{1}{2}$ same / same

b) $\frac{x}{x^3-2x+1}$ $y=0$ low / high

c) $\frac{x^3-2x+4}{x-1}$ none (✓ for slant) high / low

(12) Let $f(x) = -3x + 7$ and $g(x) = 2x^2 - 8$

a. $f(g(x)) = f(2x^2 - 8)$
$= -3(2x^2 - 8) + 7$
$= -6x^2 + 24 + 7$
$= \boxed{-6x^2 + 31}$

b. $g(f(x)) = g(-3x + 7)$
$= 2(-3x + 7)^2 - 8$
$= 2(-3x + 7)(-3x + 7) - 8$
$= 2(9x^2 - 21x - 21x + 49) - 8$
$= \boxed{18x^2 - 84x + 90}$

c. Nope

(13) $f(x) = 3x + 5$ and $g(x) = x^2$
$f(g(3)) = f((3)^2)$
$= f(9) = 3(9) + 5 = \boxed{32}$

(14) $f(x) = \dfrac{\sqrt{x-1}}{1 + \sqrt{x-1}}$ $g(h(x))$

$\boxed{g(x) = \dfrac{x}{1+x} \quad h(x) = \sqrt{x-1}}$

Answers vary

(15) $f(x) = \sqrt{\dfrac{x^3}{2}}$ $g(h(x))$

$g(x) = \sqrt{\dfrac{x}{2}}$
$h(x) = x^3$

Answers vary

(16) a) $g(x) = 4 - \frac{3}{2}x$
$f(x) = \frac{1}{2}x + \frac{3}{2}$
$g(f(x)) = g(\frac{1}{2}x + \frac{3}{2})$
$= 4 - \frac{3}{2}(\frac{1}{2}x + \frac{3}{2})$
$= 4 - \frac{3}{4}x - \frac{9}{4}$

NOT just x

$\boxed{\text{NO}}$

b) $g(x) = -\frac{2}{x} - 1$
$f(x) = \dfrac{-2}{x+1}$
$g(f(x)) = g\left(\dfrac{-2}{x+1}\right)$
$= \dfrac{-2}{\frac{-2}{x+1}} - 1$
$= \dfrac{-2(x+1)}{-2} - 1$
$= \dfrac{-2x-2}{-2} - 1$
$= \dfrac{\cancel{-2}(x+1)}{\cancel{-2}} - 1$
x ✓ $\boxed{\text{YES}}$

17 a. $f(x)=\sqrt[3]{x}-3$ D: $\mathbb{R}$ R: $\mathbb{R}$

$x=\sqrt[3]{y}-3$

$(x+3)^3=(\sqrt[3]{y})^3$

$(x+3)^3=y$

$\boxed{f^{-1}(x)=(x+3)^3}$

domain: $(-\infty,\infty)$

b. $f(x)=-4x+1$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

$x=-4y+1$

$\frac{x-1}{-4}=\frac{-4y}{-4}$

$\boxed{f^{-1}(x)=-\frac{1}{4}x+\frac{1}{4}}$

domain: $(-\infty,\infty)$

c. $f(x)=-x^2-2$ D: $x\geq 0$ R: $(-\infty,-2]$

$x=-y^2-2$

$\frac{x+2}{-1}=\frac{-y^2}{-1}$

$\sqrt{\frac{x+2}{-1}}=\sqrt{y^2}$

$\boxed{f^{-1}(x)=\sqrt{-x-2}}$

domain $(-\infty,-2]$

18 the range of $f(x)$ IS the domain of $f^{-1}(x)$