Name: _____ Period: _____

Unit 2 Review: Sequences and Series

1. Find a_2, a_3, and a_4 for the recursive sequence.
$a_n = a_{n-1} + 3$ where $a_1 = 5$ $a_2 =$ 8 $a_3 =$ 11 $a_4 =$ 14

a_2: $a_2 = a_{2-1} + 3 = a_1 + 3$
$= 5 + 3 = 8$

a_3: $a_3 = a_{3-1} + 3 = a_2 + 3$
$= 8 + 3 = 11$

a_4: $a_4 = a_3 + 3 = 11 + 3 = 14$

2. Find the sum. *Make sure you know the PROCESS, not just the shortcut!*

$\sum_{k=1}^{4} k2^k = 1(2)^1 + 2(2)^2 + 3(2)^3 + 4(2)^4$

$\boxed{98}$

3. Find the 60th term of the arithmetic sequence.
26.2, 29.9, 33.6, 37.3, ... $a = 26.2$, $d = 3.7$

$a_{60} = 26.2 + 3.7(60-1)$

$\boxed{244.5}$

4. Find the number of terms in the sequence.
3, 8, 13, ... 73 Arithmetic $a = 3$
+5 +5 a_n → find n $d = 5$

$73 = 3 + 5(n-1)$
$70 = 5n - 5$
$75 = 5n$
$15 = n$

$\boxed{15}$

5. A partial sum of an arithmetic sequence is given. Find the sum. $a = 3$ $d = 4$
3+7+11+...+39

★ Find number of terms (n)

★ Plug into $S_n = n\left(\frac{a + a_n}{2}\right)$ (we have the last term)

★ $39 = 3 + 4(n-1)$
$n = 10$

★ $S_{10} = 10\left(\frac{3+39}{2}\right)$

$\boxed{210}$

6. Determine whether the sequence is geometric, arithmetic or neither

a) 6, 24, 96, 384,... geometric, $r = 4$

b) 1.0, 1.3, 1.69, 2.197 geometric, $r = 1.3$

c) 5, 11, 13, 23, ... neither
+6 +3 +10

7. Find *nth* term and the 10th term of the geometric sequence. $a = 4$ $r = 3$
4, 12, 36, 108 ...

$a_{10} = 4(3)^{10-1} = \boxed{78732}$

$\boxed{a_n = 4(3)^{n-1}}$

8. Find the sum.

$\sum_{k=2}^{6} 2^{k-2} = 2^{2-2} + 2^{3-2} + 2^{4-2} + 2^{5-2} + 2^{6-2}$

$\boxed{31}$

9. A partial sum of an arithmetic sequence is given. Find the sum. $a = -30$ $d = +0.3$ (see #5)

-30 – 29.7 – 29.4 – ... – 0.3

★ $-0.3 = -30 + 0.3(n-1)$
$n = 100$

★ $S_{100} = 100\left(\frac{-30 + -0.3}{2}\right) = \boxed{-1515}$

10. How many terms of the series 7 + 12 + 17 + 22 + ... must be added for the sum to be 3402? S_n

$S_n = \frac{n}{2}[2a + d(n-1)]$

$3402 = \frac{n}{2}[2(7) + 5(n-1)]$

$3402 = \frac{n}{2}(9 + 5n)$

$0 = \frac{9}{2}n + \frac{5}{2}n^2 - 3402$

★ use graphing calculator
→ y = equation
→ find where y = 0 on TABLE
(x is positive & whole #)

$\boxed{36}$

$S = \frac{a}{1-r}$ if r is a fraction

11. Find the sum of the infinite geometric series, if it exists, or say diverges.

a) $\frac{2}{7} - \frac{8}{49} + \frac{32}{343} - \ldots$

$S = \frac{2/7}{1 - -4/7} = \boxed{\frac{2}{11}}$

$r = \frac{-4}{7}$, since $|r| < 1$, series converges

b) $2 + 6 + 18 + \ldots$

$r = 3$ series $\boxed{\text{diverges}}$ (∞)

c) $-a - \frac{a}{3} - \frac{a}{9} - \frac{a}{27} \ldots$

$S = \frac{-a}{1 - \frac{1}{3}} = \boxed{\frac{-3}{2}a}$

$r = \frac{1}{3}$, $r < 1$

12. Find the first term in a geometric sequence whose common ratio is 3 and whose 8th term is 8748. $r = 3$ $a_8 = 8748$ $a_n = a(r)^{n-1}$

$a_8 = a(3)^{8-1}$

$8748 = a(3)^7$

$\frac{8748}{2187} = \frac{2187a}{2187}$

$\boxed{a = 4}$

13. How many terms are there in the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \frac{1}{1024}$? $a = 1$ $r = 1/2$

$\boxed{11}$

$\frac{1}{1024} = 1\left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{n-1} \Rightarrow 10 = n-1$

14. Find the sum of the first 8 terms of the sequence $1 + 4 + 16 + \ldots$ $S_n = a\left(\frac{1-r^n}{1-r}\right)$

$a = 1$ $r = 4$

$S_8 = 1\left(\frac{1-4^8}{1-4}\right) = \boxed{21845}$

15. For what real value of c will $6, 2, c$ be consecutive terms in a geometric sequence?

needs a constant multiplier (divider)

$\frac{c}{2} = \frac{2}{6}$

$\frac{6c}{6} = \frac{4}{6}$

$c = 4/6$

$\boxed{2/3}$

16. Evaluate $\binom{15}{3} = \frac{15!}{3!(15-3)!}$ OR ${}_{15}C_3$

$\boxed{455}$

17. Expand $(2x-3)^5$

$= \binom{5}{0}(2x)^5(-3)^0 + \binom{5}{1}(2x)^4(-3)^1 + \binom{5}{2}(2x)^3(-3)^2$
$+ \binom{5}{3}(2x)^2(-3)^3 + \binom{5}{4}(2x)^1(-3)^4 + \binom{5}{5}(2x)^0(-3)^5$

$= 1(32)x^5 + 5(16)(-3)x^4 + 10(8)x^3(9)$
$+ 10(4)x^2(-27) + 5(2)x(81) + 1(-243)$

$\boxed{32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243}$

18. Find the 4th term in the expansion $(4x-2y)^8$.

$\binom{8}{3}(4x)^5(-2y)^3$ 0+0 add to 8

$n-1$ matches

$56(1024)x^5(-8)y^3$

$\boxed{-458752x^5y^3}$

19. Write the term that contains x^6 in the expansion $(x+2y)^{10}$.

$\binom{10}{4}(x)^6(2y)^4$

$210x^6(16)y^4$

$\boxed{3360x^6y^4}$

20. In an arithmetic sequence $a_2 = 4x + y$ and $a_3 = 6x + 5y$. Find a_{11}. $a_1 = 2x - 3y$

$d = +2x + 4y$

$a_{11} = a_1 + d(11-1)$

$a_{11} = (2x - 3y) + (2x + 4y)(10)$

$= 2x - 3y + 20x + 40y$

$\boxed{22x + 37y}$