

Name: \_\_\_\_\_

### Fall Semester Exam Review

A completed review will be worth the curve on the exam. **Questions with \*\*\* are calculator questions**



#### Unit 1 Polynomials

Graph (without a calculator)

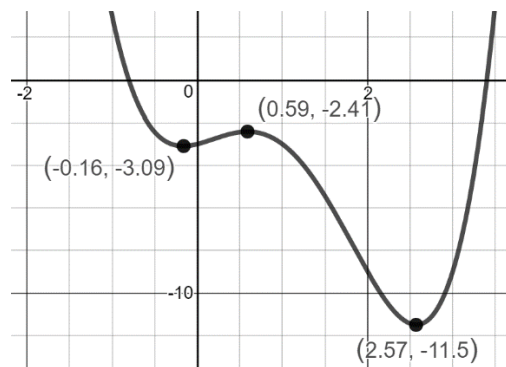
1.  $y = -x(x+3)^2$

2.  $y = (2x-1)^2(x+5)^2$

3.  $y = -x^3(x-3)^2(x+5)$

4. Find the remainder when the polynomial  $2x^4 + x^3 - x + 2$  is divided by the binomial  $x - 2$

5. Find the relative extrema of the function  $y = -x(x+3)^2$  and list the intervals where the function is increasing and decreasing



#### Unit 2 Rational Functions

Find all asymptotes and graph

6.  $\frac{2x-3}{x-4}$

7.  $\frac{x^2-1}{x^2+11x-12}$

8.  $\frac{x+6}{x^2-9}$

9.  $\frac{x^2-2x-15}{x+3}$

10.  $\frac{x^2+1}{x-3}$

Find the domain of the function

11.  $\frac{2}{x^2-4x+3}$

12.  $\frac{2}{3x^2-7x+6}$



### Unit 3 Logs and Exponents

Write as a single log

13.  $\log 25 + \log 4$

14.  $6(\log a + \log b)$

15.  $\ln 2x + 3(\ln x - \ln y)$

Solve. Leave answers in terms of  $\ln$  if needed

16.  $5^x = \frac{1}{25}$

17.  $3^x = 8$

18.  $27^{2x-1} = 9^{x+3}$

19.  $\log_4(3x-2) = 2$

20.  $\log_8 x = \frac{2}{3}$

21.  $5\log_2(\log_3 81)$

22.  $5e^x - 12 = 7$

23.  $\ln 1$

24.  $\ln e^3$

25.  $2 = \log_3(9n+10) - \log_3(5n)$

26.  $\ln 5 + \ln(x+2) = \ln 7$

27.  $\log x + \log(x-9) = 1$

\*\*\*28. A radioactive substance has a half-life of 420 years. How much remains of a 2 oz. sample after 200 years?

\*\*\*29. A bacteria culture has an initial count estimate of 4000. After 20 minutes the count is 22,400. What is the growth rate, and approximately how many minutes did it take for the culture to double?

\*\*\*30. \$3000 is invested in an account that pays 5% annual interest. How much more money would you make if the interest was compounded daily for 10 years instead of compounded quarterly?

\*\*\*31. How many years will it take an investment of \$1000 to double itself when interest is compounded continuously at 6% annually?



\*\*\*Unit 4 Series and Sequences

32. Write the first 5 terms of the sequence whose nth term is  $a_n = n^2 - 3$

33. If  $a_n = a_{n-1} + 5$  and  $a_1 = 3$ , find  $a_2$ ,  $a_3$  and  $a_4$

34. Evaluate  $\sum_{x=5}^9 |4 - x|$

35. Evaluate  $\sum_{c=0}^4 (-2)^c$

36-37: Rewrite using sigma notation:

36.  $-4 - 8 - 12 - 16 - 20$

37.  $12 + 3 - 6 - 15$

Determine whether the sequence is arithmetic, geometric or neither. If arithmetic or geometric, find the nth term.

38. 5, 7, 9, 11, 13...

39.  $1/7$ , 1, 7, 49...

40. 15, 17, 20, 23, 25...

Find the sum of the series

41.  $\frac{1}{3} - \frac{2}{3} + \frac{4}{3} - \frac{8}{3} \dots$

42.  $25 + 5 + 1 + \dots$

43. Expand the following binomial  $(x - 2y)^5$

44. Find the 3<sup>rd</sup> term of the expansion of  $(2a + 3b)^{12}$



Unit 5 Conics

Is NOT covered on either part of the exam. (phew!!)



Unit 6 Right Triangle Trigonometry **PREAP PORTION ONLY**

51. Find the exact values of the six trig functions of an angle  $\theta$  whose terminal side passes through the point  $(-5, -7)$ .

Find the exact value:

52.  $\cos \frac{\pi}{4} \sin \frac{7\pi}{6} - \sin \frac{\pi}{6} \cos \frac{3\pi}{4}$

53.  $\cot^2 \frac{11\pi}{6} - \csc^2 \frac{11\pi}{6}$

54.  $\sin \frac{\pi}{2} + 6 \cos \frac{\pi}{3}$

55.  $\frac{\cos \frac{5\pi}{3}}{\sin \frac{5\pi}{3}}$

56.  $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6} + \tan^2 \frac{\pi}{6} - \sec^2 \frac{\pi}{6}$

57. Convert  $40^\circ$  to radians

58. Convert  $\frac{\pi}{9}$  radians to degrees

59. Find a positive and negative angle coterminal to  $84^\circ$

60. If  $\sin \theta < 0$  and  $\cot \theta < 0$ , then  $\theta$  must lie in which quadrant?

\*\*\*61. Your cat is trapped on a tree branch 6.5 meters above the ground. Your ladder is only 6.7 meters long. If you place the ladder's tip on the branch, what angle will the ladder make with the ground?

\*\*\*62. Commercial airliners fly at an altitude of about 10 kilometers. They start descending toward the airport when they are far away, so that they will not have to dive at a steep angle.

- If the pilot wants the plane's path to make an angle of  $3^\circ$  with the ground, at what horizontal distance from the airport must he start descending?
- If he starts descending a ground distance of 300 km from the airport, what angle will the plane's path make with the horizontal?

\*\*\*63. While standing on a cliff 120 feet high, I see a sailboat at an angle of depression of  $21^\circ$ . What is the horizontal distance between the cliff and the sailboat?



### Unit 7: Graphing Trigonometric Functions

Graph 1 cycle. Identify the period, amplitude, vertical shift and phase shift:

65.  $y = -10 + 20 \sin 2\left(x - \frac{\pi}{8}\right)$

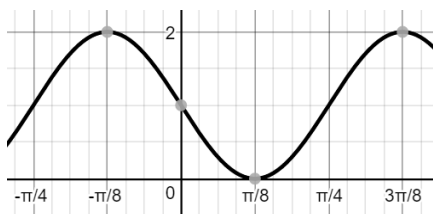
66.  $y = -5 \cos \frac{1}{2}(x + \pi) + 3$

67.  $y = 3 + 2 \cos \frac{1}{5}(x - \pi)$

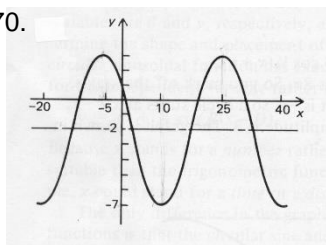
68.  $y = 2 - 6 \sin \frac{\pi}{4}(x - 1)$

Write an equation for each graph as both sine and cosine:

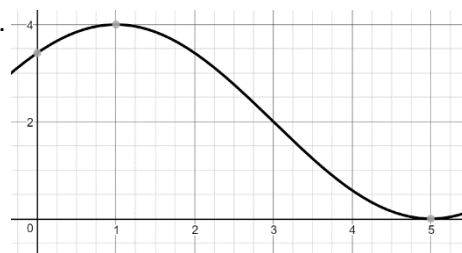
69.



70.



71.



72. \*\*\*Find each value correct to 3 decimal places.

$$f(x) = 5 + 2\cos\frac{\pi}{4}(x-10)$$

a.)  $f(17.3)$

b.) Find the first three positive values for  $x$  if  $f(x) = 6.7$

c.) What is the first positive  $x$  value that gives the maximum  $y$  value?

73. \*\*\*Researchers find a creature from an alien planet. Its body temperature is varying sinusoidally with time. 35 minutes after they start timing, it reaches a high of  $120^{\circ}F$ . 20 minutes after that it reaches its next low,  $104^{\circ}F$ .

a. Sketch a graph of this sinusoid.

b. Write an equation expressing temperature in terms of minutes since they started timing.

c. What was the temperature when they first started timing?

d. Find the first 3 times after they started timing at which the temperature was  $114^{\circ}F$ .

74. \*\*\* A spacecraft is in an elliptical orbit around the earth. At time  $t=0$  hours, it is at its apogee (highest point)  $d=1000$  km above the earth's surface. Fifty minutes later, it is at its perigee (closest point)  $d=100$  km above the surface. Round to three decimal places.

a.) Write an equation for  $d$  in terms of  $t$ .

b.) Predict the first 3 positive values for  $t$  which the spacecraft is 200 km from the earth.

c.) In order to transmit back to earth, the spacecraft must be within 700 km of the surface. For how many consecutive minutes will the spacecraft be able to transmit?

75. Graph the parent function for each

a.  $y = \tan x$

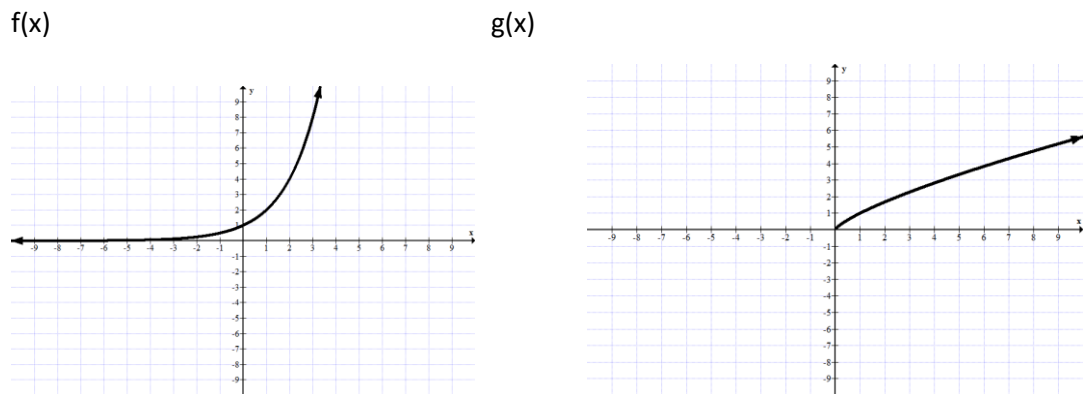
b.  $y = \cot x$

c.  $y = \csc x$

d.  $y = \sec x$

All of the district portion will be multiple choice, with calculator allowed. Check your answers on mskmath.com!

1. The graph of  $f(x)$  and  $g(x)$  is shown below.



Fill out the following table:

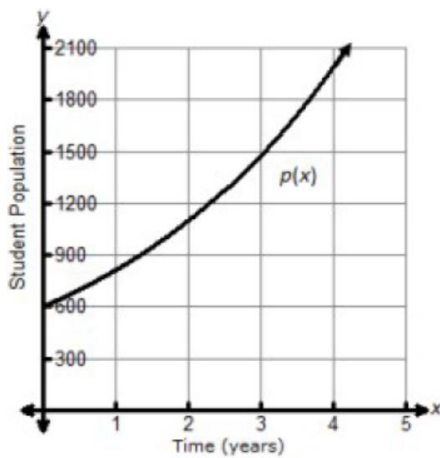
	$f(x)$	$g(x)$
Is the function continuous?		
Asymptotes		
Domain		
Increasing or Decreasing?		

2. A finite series is shown below. What is the sum?

$$\sum_{n=1}^4 (n^3 - 1)$$

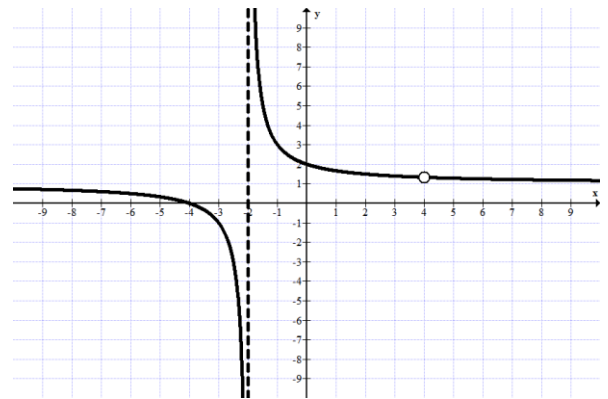
3. Given that  $f(x)=3^{2x}$  and  $g(x)=9^x$ , graph the functions to determine the relationship between  $f(x)$  and  $g(x)$ .
4. Westin purchased a piece of land in the shape of a right triangle on which to plant an apple orchard. On the first row of trees Westin planted 20 trees. Each subsequent row contained 2 less trees. How many apple trees would be planted on the sixth row?
5. The free-fall speed of an object, in terms of distance, measured in meters, can be modeled by the function  $s(d) = 4d^{\frac{1}{2}}$ . If the free-fall speed is measured at 5.657 meters/second, approximately how far has the object fallen?
6. Find the inverse for the function  $f(x) = (x - 2)^3 + 1$

7. A new high school starts with a population of 600 freshmen and sophomore students. Each year, the population increases by 35% per year. The school's population can be modeled by the function  $p(x) = 600(1.35)^x$ , where  $x$  represents time in years and  $p(x)$  represents population of students.



Describe the end behavior of the function.

8. Describe the discontinuities for the graph of the function shown.



9. An algebraic expression involving logarithms is shown below. Condense to a single log.

$$2\log(x-2) - \frac{1}{2}\log(x+2) + 6\log(x-1)$$

10. The equation of a rational function is shown below.

$$f(x) = \frac{x^2 - 16}{x^2 - 6x + 8}$$

Describe the left-sided behavior and right-sided behavior of the rational function as  $x \rightarrow 4$ .

11. Which series of function compositions can be used to represent  $f(x) = \frac{x^2 + 4}{x^2 + 1}$ ?

$$g(x) = \frac{x}{x-3}$$

$$h(x) = x + 4$$

$$j(x) = x^2$$

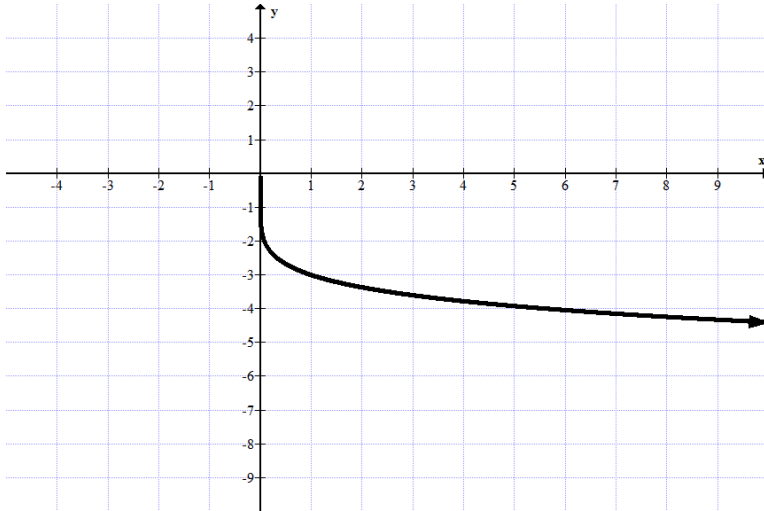
- A.  $f(x) = h(j(x))$     B.  $f(x) = g(h(j(x)))$     C.  $f(x) = j(h(g(x)))$     D.  $f(x) = g(j(h(x)))$



12. Write a rational function that has both a vertical and an oblique asymptote.

13. The cost to inoculate  $x\%$  of a population from a single strain of flu virus in billions of dollars,  $C$ , is given by the formula  $C(x) = \frac{320}{100 - x}$ . If the CDC has a budget of 7 billion dollars to spend on inoculations, then what is the maximum percentage of the population that it can afford to inoculate, rounded to the nearest hundredth?

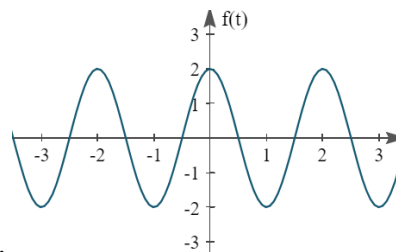
14. Given the graph of the power function,  $f(x) = -3x^{\frac{1}{6}}$ , describe the end behavior of the graph.



15. Given the polynomial function  $f(x) = \frac{1}{3}(x+4)^2 - 5$ , describe the transformations of the parent function.

Write "none" if the transformation does not apply.

- Vertical Shift:
- Horizontal Shift:
- Vertical Compression/Stretch:
- Horizontal Compression/Stretch:



16. Find and justify the symmetry of the graph shown.

17. The price per unit,  $p(q)$ , of a popular copy machine, in terms of the quantity of copy machines demanded,  $q$ , is given by the formula  $p(q) = 1500 - 150\ln(q)$ . Predict the number of copy machines demanded if the price per unit is \$500.

18. Graph the function  $f(x) = -2x^3 - 2x^2 + x + 3$ . List the domain and range and where the function is increasing and/or decreasing.

19. Find a function for which an inverse function does not exist.

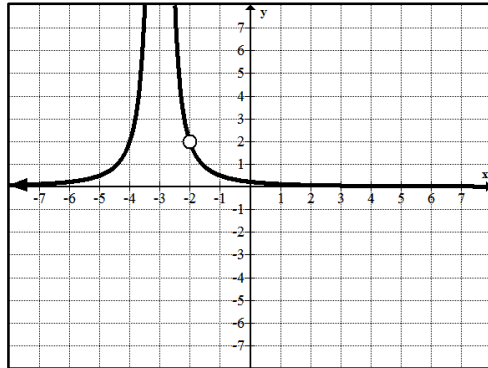
20. The weight of a radioactive material in grams,  $w$ , over a period of weeks,  $t$ , is given by the table shown below.

$t$	0	1	2	3	4	5
$w$	50	47	43	40	37	35

Using the table of values, which equation best represents the data?

- A.  $w = 50 - 3t$       B.  $w = 50t^{0.93}$       C.  $w = 50(0.93)^t$       D.  $w = 50 + (0.93)^t$

21. Describe the following behavior.



- Right side behavior as  $x \rightarrow -2, f(x) \rightarrow \underline{\hspace{2cm}}$
- Left side behavior as  $x \rightarrow -2, f(x) \rightarrow \underline{\hspace{2cm}}$

22. List all solutions to the equation  $x^3 - 7x^2 + 12x = 0$ .

23. If  $f(x) = \log x$ , list all of the transformations for the function  $f(0.5(x - 3)) + 1$ .

- a. Vertical Shift:
- b. Horizontal Shift:
- c. Vertical Compression/Stretch:
- d. Horizontal Compression/Stretch:

24. A new employee earns \$53,000 during his first year of work and receives a 2% raise each year. Write a sigma notation that could be used to determine the total amount earned by this employee over the first 10 years. Remember, it's a RAISE, he doesn't lose the money!

25. The equation for a rational function is given below.

$$f(x) = \frac{2}{x+3} - 4$$

Fill in the following information:

- a. Domain:
- b. Range:
- c. Horizontal asymptote:
- d. Vertical asymptote:
- e. Is the function increasing or decreasing?