8.2 Polynomial Operations

Warm-up Monday

Without a calculator, find the quotient and remainder of the following problem.

$$8954762 \div 23$$

389,337 R11

23) 8954762
-69
205
-184
214
. . . .
172
-161
11

About Me

1. Rank the following: Chipotle, Freebirds, Qdoba, some other burrito place I don't know about...

2. What's the most interesting fact you know?

8.2 Polynomial Operations

EQ: How do I determine the degree of a polynomial?

A **term** is part of an algebraic expression that can be written using constants, variables, multiplication and division. The constants are called coefficients. $3x$ A **polynomial** can be written using terms and addition and subtraction. The term of the polynomial which does not include a variable is called the constant term. Any letter may be used as the variable in a polynomial.

Note the characteristics of a polynomial.

- All exponents are whole numbers.
- No variables in the denominator. (includes negative exp)
- No variables under a radical. (fraction exponents)

Any letter may be used as the variable in a polynomial. Examples of **polynomials** include the following.

POLYNOMIALS	NOT POLYNOMIALS
$2x^2$ 5 x^5+1 x^2-4x+5 $3x^2+\sqrt{5}$	$1.5x^{-2} = \frac{1.5}{x^2}$ $3x^2+\sqrt{x}+5$

8.2 Polynomial Operations

EQ: How do I determine the degree of a polynomial?

Degree of a Polynomial – The *exponent* of the highest power of x is the **degree** of the polynomial, and the coefficient of this highest power of the variable is the **leading coefficient**.

Polynomial	Degree	Leading Coefficient	Constant Term
$\boxed{6x^7} + 4x^3 + 5x^2 - 7x + 10$	7	6	10
$1x^3 + 0$	3	1	0
$12x^0$	0	—	12
$2x^6 + 3x^7 \boxed{-x^8} - 2x - 4$	8	-1	-4
$(x^1+2)(x-1)^2$ FOIL ☹	3	1	2(-1)(-1) 2

factored → add exp.

8.2 Polynomial Operations

EQ: How do I determine the degree of a polynomial?

Polynomial functions of degree less that 5 are often referred to by special names.

- First-degree polynomial functions are called ___linear___ **functions.** x^1+2
- Second-degree polynomial functions are called ___quadratic___ **functions.** x^2+x+2
- Third- degree polynomial functions are called ___cubic___ **functions.**
- Fourth- degree polynomial functions are called ___quartic___ **functions.**

8.2 Polynomial Operations

EQ: How do I determine the degree of a polynomial?

Adding and Subtracting Polynomials To add or subtract polynomials, combine like terms

Distribute negative when subtracting!

ex. $(-2x^3 + x^2 - 4x + 1) - (2x^3 + x + 4)$

$$\boxed{-4x^3 + x^2 - 3x - 3}$$

8.2 Polynomial Operations

EQ: How do I determine the degree of a polynomial?

Multiplying Polynomials To multiply polynomials, distribute/FOIL (box)

ex. $(2x-3)(x^2+3x-5)$

$2x^3+6x^2-10x-3x^2-9x+15$

$\boxed{2x^3+3x^2-19x+15}$

8.2 Polynomial Operations

EQ: How do I determine the degree of a polynomial?

Dividing Polynomials *put zeros for missing

Ex. $(3x^4 - 8x^2 - 11x + 1) \div (x - 2)$

Synthetic Division

Only works when divisor is first degree binomial

(x-2)

Synthetic basics

1. Bring down 1st #
2. Multiply
3. Add down column
4. Repeat for each column
5. Answer is one degree less

Don't forget the zeros for missing terms!

* opposite sign outside
* coefficients inside

$$\begin{array}{r|rrrrr} 2 & 3 & 0 & -8 & -11 & 1 \\ & \downarrow & 6 & 12 & 8 & -6 \\ \hline & 3x^3 & +6x^2 & +4x & -3 & \boxed{-5} \end{array}$$

remainder

Long Division

$\frac{3x^4}{x} \to 3x^3$

$$\begin{array}{r} 3x^3 + 6x^2 + 4x - 3 \\ x-2 \overline{) 3x^4 + 0x^3 - 8x^2 - 11x + 1} \\ -(3x^4 - 6x^3) \\ \hline 6x^3 - 8x^2 \\ -(6x^3 - 12x^2) \\ \hline 4x^2 - 11x \\ -(4x^2 - 8x) \\ \hline -3x + 1 \\ -(-3x + 6) \\ \hline R\ -5 \end{array}$$

$\boxed{3x^3 + 6x^2 + 4x - 3 \quad R - 5}$

to check your answer...

answer * divisor + remainder = original

8.2 –Operations with Polynomials evens ☺ **Name** ____________________

In Exercises 1 –8 determine whether the given algebraic expression is a polynomial. If it is, list its leading coefficient, constant term, and degree.

1. $1+x^3$

2. 7^x+2x+1

3. $(x+\sqrt{3})(x-\sqrt{3})$

4. $4x^2+3\sqrt{x}+5$

5. $\frac{7}{x^2}+\frac{5}{x}-15$

6. $(x-1)^k$

where k is a fixed, positive integer

In Exercises 7 – 14 perform the indicated operations

7. $(m^2+3)-(4-3m)$

8. $(2x^2-4x+7)-(-2x^2+3x-7)$

9. $5a^4(a^2-4a+3)$

10. $(x+2)(x^2-4x+5)$

11. $(7x-3)^2$

12. $(5-2x)^2$

13. $(2x+5)(2x-5)-(2x+5)^2$

14. $(x+3)^2+(x-3)^2$

In Exercises 15 – 19, use synthetic division to find the quotient and remainder.

15. $(3x^4 - 8x^3 + 9x + 5) \div (x - 2)$

16. $(4x^3 - 3x^2 + x + 7) \div (x - 2)$

17. $(3x^3 - 2x^2 - 8) \div (x + 5)$

18. $(2x^4 + 5x^3 - 2x - 8) \div (x + 3)$

19. $(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1) \div (x - 1)$

In Exercises 20 – 22, state the quotient and remainder when the first polynomial is divided by the second using long division. Check your division by calculating: (Divisor)(Quotient) + Remainder.

20. $3x^4 + 2x^2 - 6x + 1; \quad x + 1$

21. $3x^4 - 3x^3 - 11x^2 + 6x - 1; \quad x^3 + x^2 - 2$

22. $x^5 - 1; \quad x - 1$