8.4 FACTOR & REMAINDER THEOREMS

WARM-UP THURSDAY

Factor the following polynomials and find all ~~solutions.~~

$$\boxed{\left\{-\frac{4}{3}, -3, 3\right\}}$$

$g(x) = x^2 - 5x - 6$ X

$g(x) = (x-6)(x+1)$

$\boxed{\{6, -1\}}$

$x - 6 = 0$ $\quad x + 1 = 0$

$x = 6$ $\quad x = -1$

$f(x) = (3x^3 + 4x^2)(-27x - 36)$

$x^2(3x+4) - 9(3x+4)$

$(3x+4)(x^2-9)$

$(3x+4)(x+3)(x-3)$

$3x + 4 = 0$ $\quad x + 3 = 0$

$\frac{3x}{3} = \frac{-4}{3}$ $\quad x - 3 = 0$

ABOUT ME:

1. Would you rather play a villain or a hero in a movie?

2. Cookies or cake?

8.4 FACTOR & REMAINDER THEOREMS

EQ: How do I use the factor and remainder theorems to simplify polynomials?

REMAINDER THEOREM:

If P(x) is divided by x−c, then the remainder is equal to P(c).

plug in c (change sign) to equation, gives remainder

ex. What is the remainder when P(x) is divided by f(x)?

A. $P(x)=2x^3-7x^2+5$ $f(x)=x-\boxed{3}$

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 5 \\ & \downarrow & 6 & -3 & -9 \\ \hline & 2 & -1 & -3 & \boxed{-4} \end{array}$$

$P(3)=2(3)^3-7(3)^2+5$

$P(3)=-4$ ☺

B. $P(x) = x^{79} + 3x^{24} + 5$ $f(x) = x+1$

even power → +
odd power → −

$-1 \,|\, 1\ 0\ 0\ 0\ 0\ 0\ 0$ ☹

$P(-1)= (-1)^{79} + 3(-1)^{24} + 5$

$-1 + 3 + 5 = \boxed{7}$

C. $P(x) = 3x^4 - 8x^2 + 11x + 5$ $f(x) = x+2$

$P(-2)= 3(-2)^4 - 8(-2)^2 + 11(-2) + 5 = \boxed{-1}$

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & -8 & 11 & 5 \\ & \downarrow & -6 & 12 & -8 & -6 \\ \hline & 3 & -6 & 4 & 3 & \boxed{-1} \end{array}$$

8.4 FACTOR & REMAINDER THEOREMS

EQ: How do I use the factor and remainder theorems to simplify polynomials?

FACTOR THEOREM:

c is a zero of P if and only if x−c is a factor of P(x).

A remainder of zero means you have a factor!

ex. Show that x−3 is a factor of $f(x) = x^3 - 4x^2 + 2x + 3$

x−c

$f(3) = 3^3 - 4(3)^2 + 2(3) + 3$

$f(3) = 0$ ✓

ex. $P(x) = x^3 - 7x + 6$

$0x^2$

Show that 1 is a zero and factor completely.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

$(x-1)(x^2+x-6)$

-6

3 -2

1

$$\boxed{(x-1)(x-2)(x+3)}$$

8.4 FACTOR & REMAINDER THEOREMS

EQ: How do I use the factor and remainder theorems to simplify polynomials?

FINDING A SPECIFIED POLYNOMIAL...

ex. Find a polynomial with the degree n and the given zeros.

A. n =4 and zeros are -6, -5, 1, 2

$\boxed{f(x)=(x+6)(x+5)(x-1)(x-2)}$

degree = 4 :)

B. n = 3 and the zeros are -5, 2

$f(x)= (x+5)(x-2)^2$

OR

$(x+5)^2(x-2)$

8.4 –Remainder and Factor Theorems odds **Name** ______________

Find the zeros of the given polynomials algebraically by factoring and setting each factor equal to zero. Check your answers with your calculator.

1. $f(x)=x^2+7x$ **2.** $g(x)=x^2-5x-6$ **3.** $h(x)=6x^2+x-1$

4. $f(a)=(a+3)^2-64$ **5.** $g(t)=3t^3-27t$ **6.** $k(y)=9y^4+3y^3-6y^2$

7. $h(x)=x^3-5x^2-x+5$ **8.** $f(x)=3x^3+4x^2-27x-36$ **9.** $k(x)=8x^3-12x^2-6x+9$

Find the remainder when $f(x)$ is divided by $g(x)$, without using division.

10. $f(x)=x^{10}+x^8$; $g(x)=x-1$

11. $f(x)=x^6-10$; $g(x)=x-2$

12. $f(x)=3x^4-6x^3+2x-1$; $g(x)=x+1$

13. $f(x)=x^3-2x^2+5x-4$; $g(x)=x+2$

Use the Factor Theorem to determine whether $h(x)$ is a factor of $f(x)$.

14. $h(x)=x-1$ $f(x)=x^5+1$ **15.** $h(x)=x-\frac{1}{2}$ $f(x)=2x^4+x^3+x-\frac{3}{4}$

16. $h(x)=x+2$ $f(x)=x^3-3x^2-4x-12$ **17.** $h(x)=x+1$ $f(x)=14x^{99}+65x^{56}-51$

Find a polynomial with the given degree, *n*, the given zeros, and no other zeros.

18. $n = 3$; zeros 1, 7, -4

19. $n = 3$; zeros 1, -1

20. $n = 5$; zero 2

21. Find a polynomial function *g* of degree 3 such that $g(10) = 17$ and the zeros of $g(x)$ are 0, 5, and 8.

22. Find a polynomial function *f* of degree 4 such that $f(3) = 288$ and the zeros of $f(x)$ are 0, -1, 2, and -3.

23. Find a number *k* such that $x + 2$ is a factor of $x^3 + 3x^2 + kx - 2$.

24. Find a number *k* such that $x - 1$ is a factor of $k^2x^4 - 2kx^2 + 1$.

25. When $x^3 + cx + 4$ is divided by $x + 2$, the remainder is 4. Find *c*.