

How do I find the zeros for a polynomial function?

KEY

I probably should have taken a 2 out first...

Example

$$f(x) = 2x^3 + 3x^2 - 8x + 3$$

$$\pm 1, \pm 2 \quad \pm 1, \pm 3$$

1<sup>st</sup> Find all the factors of the constant and of the leading coefficient.

2<sup>nd</sup> Form all rational numbers by making fractions with the numerator a factor of the constant and the denominator a factor of the leading coefficient.

$$\pm \left\{ 3, 1, \frac{3}{2}, \frac{1}{2} \right\}$$

3<sup>rd</sup> Using synthetic substitution, find one solution to the problem.

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & \downarrow & & & \\ & 2 & 5 & -3 & 0 \end{array} \quad \text{☺}$$

4<sup>th</sup> Rewrite the function as two factors.

$$(x-1)(2x^2+5x-3)$$

$$\downarrow \quad \downarrow$$

$$\pm 1, \pm 2 \quad \pm 1, \pm 3$$

5<sup>th</sup> Repeat this process as needed to get all your factors of the polynomial.

$$\begin{array}{r|rr} -3 & 2 & 5 \\ & \downarrow & \\ & -6 & 3 \end{array}$$

$$(x-1)(x+3)(2x-1)$$

6<sup>th</sup> Set each factor equal to zero and solve the resulting equations.

$$\begin{aligned} x-1 &= 0 \\ x &= 1 \\ x+3 &= 0 \\ x &= -3 \\ 2x-1 &= 0 \\ x &= \frac{1}{2} \end{aligned}$$

$$\left\{ -3, \frac{1}{2}, 1 \right\}$$

Your Turn

$$f(x) = 2x^4 - 2x^3 + 2x^2 - 6x - 12$$

$$\pm 1, \pm 2 \quad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$\pm \left\{ 1, 2, 3, 4, 6, 12, \frac{1}{2}, \frac{3}{2} \right\}$$

$$\begin{array}{r|rrrrr} 1 & 2 & -2 & 2 & -6 & -12 \\ & \downarrow & & & & \\ & 2 & 0 & 2 & -4 & \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 2 & -2 & 2 & -6 & -12 \\ & \downarrow & & & & \\ & 2 & 2 & 6 & 6 & 0 \end{array} \quad \text{☺}$$

$$(x-2)(2x^3+2x^2+6x+6)$$

$$2(x-2)(x^3+x^2+3x+3)$$

$$x^2(x+1)+3(x+1)$$

$$2(x-2)(x+1)(x^2+3)$$

$$\begin{aligned} x-2 &= 0 \\ x &= 2 \\ x+1 &= 0 \\ x &= -1 \\ x^2+3 &= 0 \\ x^2 &= -3 \\ \text{ONE} \end{aligned}$$

$$\left\{ 2, -1 \right\}$$