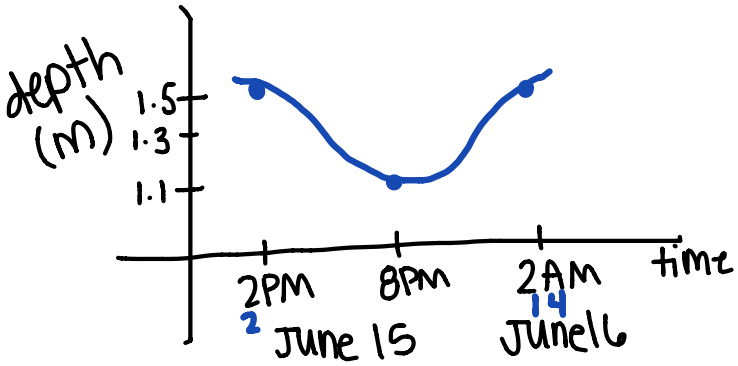
4.3 Sinusoidal Functions as Mathematical Models (2) Name:_____

1. You are on the beach in Galveston, Texas. At 2:00 pm on June 15th, the tide is in (the water level is at its deepest). At that time you find that the depth at the end of the pier is 1.5 meters. At 8:00 pm the same day when the tide is out, you find that the depth of the water is 1.1 meters. Assume that the depth of the water varies sinusoidally with time.
   a. Write an equation expressing the depth of the water in terms of the number of hours that have elapsed since 12:00 noon on June 15th.

12 hours to complete cycle



$A = 0.2$ $B = \frac{2\pi}{12} = \frac{\pi}{6}$

$C = 1.3$ $D = \cos 2$ or 12

0 = noon June 15, 13 = 1 AM June 16th

   b. What will the depth of the water be at 4:00 pm on June 15th?

$x = 4$ 1.4m

   c. What will the depth of the water be at 7:00 am on June 16th? (First find how many hours have elapsed since 12 noon on June 15th.)

$x = 19$ 1.13m

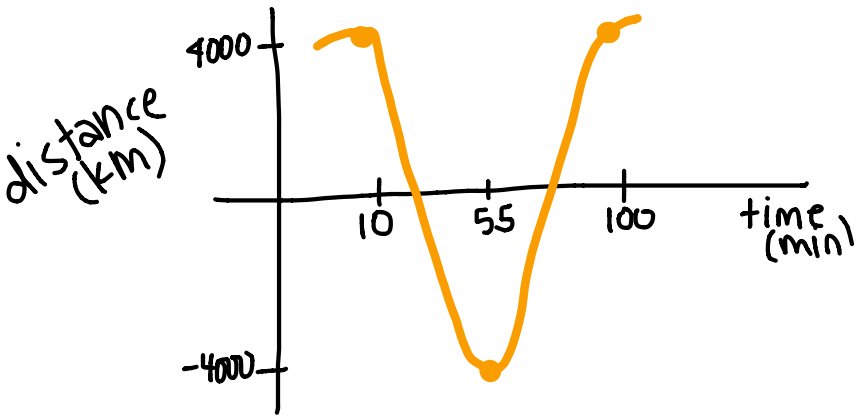
   d. At what time will the first low tide occur on June 16th?

$y = 1.1$ after 12

$x = 20 \rightarrow$ 8:00AM

2. When a spaceship is fired into orbit from a site such as Cape Canaveral, which is not on the equator, it goes into an orbit that takes it alternatively north and south of the equator. It's distance from the equator is a sinusoidal function of time. Suppose that Discovery is launched from Cape Canaveral. 10 minutes after lift-off, it reaches its farthest distance north of the equator, 4000 kilometers. Half a cycle later it reaches its farthest distance south of the equator on the other side of the earth, also 4000 km. The Discovery completes an orbit every 90 minutes. Let y be the number of kilometers the space shuttle is north of the equator. (Consider distances south of the equator to be negative.) Let t be the number of minutes since lift-off.

   a. Sketch a graph of this situation.



$A = 4000$

$C = 0$

$B = \frac{2\pi}{90} = \frac{\pi}{45}$

D: cos 10

   b. Write an equation expressing y in terms of t.

$y = 4000\cos\frac{\pi}{45}(t-10)$

   c. What is the space ship's distance from the equator when $t = 163$. Is the space shuttle north or south of the equator at this time?

1236.01 km South

   d. Calculate the distance of Cape Canaveral from the equator by finding y when $t = 0$.

3064.18 km north

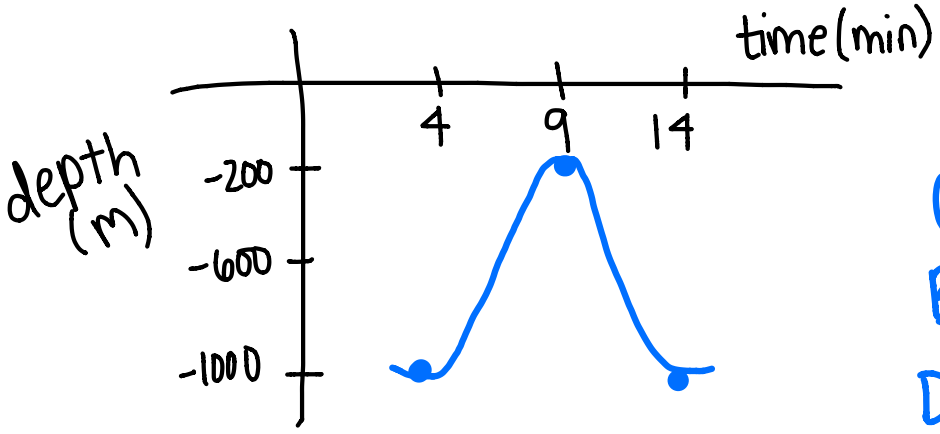
   e. What is first time at which Discovery is 1600 km south of the equator?

$y = -1600$

38.39 min

3. You are on board a submarine, submerged in the Pacific Ocean. At time $t = 0$ you make contact with an enemy destroyer. Immediately you start porpoising (going deeper and shallower). At time $t = 4$ minutes you are at your deepest, 1000 meters **below** the surface. (Let the surface of the water be $y = 0$.) At time $t = 9$ minutes you are at your next shallowest point, 200 meters below the surface. Your depth varies sinusoidally with time.

   a. Sketch a graph of your depth with respect to time.



$A = 400$

$C = -600$

$B = \frac{2\pi}{10} = \frac{\pi}{5}$

D: cos 9
- cos 4 or 14

   b. Write an equation expressing your depth in terms of time.

$y = -600 + 400 \cos \frac{\pi}{5}(x - 9)$

   c. Your submarine is safe from the enemy destroyer when it is 300 meters or more below the surface. Was your submarine safe at time $t = 0$? Justify your answer.

NO, at X=0, the submarine was at 276.4m.

   d. Between what two times is your submarine first safe?

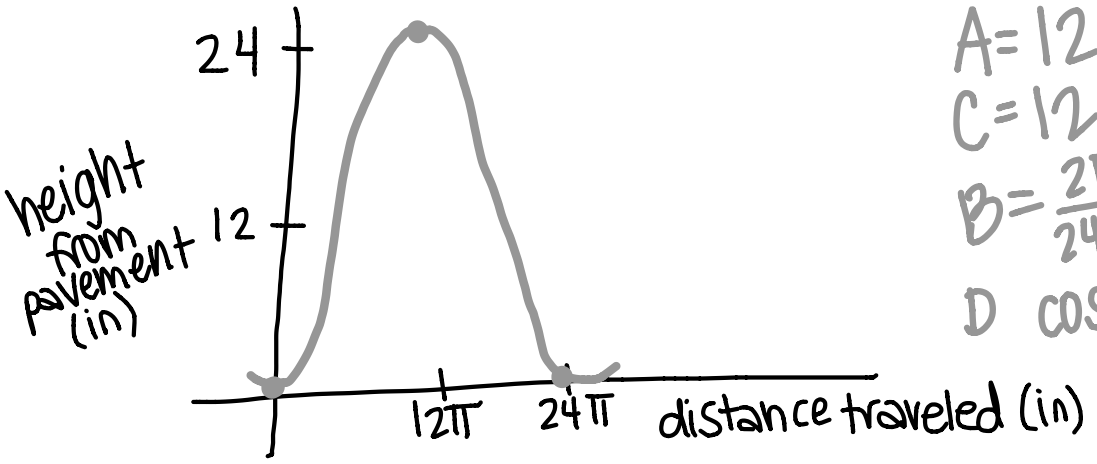
$y = -300$

between 0.15 and 7.85 min

4. As you stop your car at a traffic light, a pebble becomes wedged between the tire treads. When you start off, the distance of the pebble from the pavement varies sinusoidally with the distance you have traveled. The period is the circumference of the wheel, and the diameter of the wheel is 24 inches.

$C = 2\pi r$ or πd

a. Sketch a graph of this function.



$A = 12$

$C = 12$

$B = \frac{2\pi}{24\pi} = \frac{1}{12}$

D cos 12π

b. Write an equation of this function.

$y = 12 + 12\cos\frac{1}{12}(x - 12\pi)$

c. Predict the pebble's distance from the pavement when you have driven 35 inches.

$x = 35$

23.7 in

d. What are the first two distances you have traveled when the pebble is 11 inches from the pavement?

$y = 11$ 17.85in & 57.55in