Name: KEY

PreAP PreCalculus

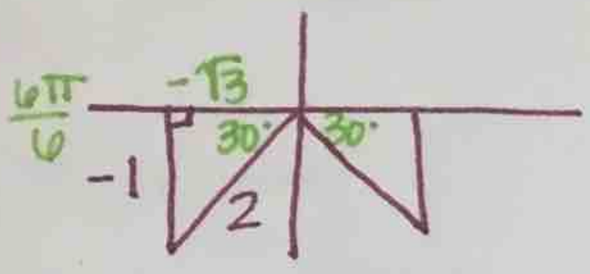
S $\frac{O}{H}$ C $\frac{A}{H}$ T $\frac{O}{A}$

Sin⊕ | All⊕
Tan⊕ | Cos⊕

Unit 3 Test Review

Find the value of θ where $0 \le \theta < 2\pi$ (95% of the time → 2 answers)

1. $\sin^{-1}\left(-\frac{1}{2}\right) = \theta$ $\frac{\pi}{6}$ ref ∠



$\boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$

2. $\cos^{-1}(-1) = \theta$

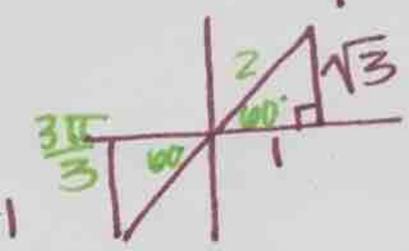
NO △ ∴

	S	CT
0	0	1 0
$\pi/2$	1	0 U
π	0	-1 0
$3\pi/2$	-1	0 U

$\boxed{\pi}$

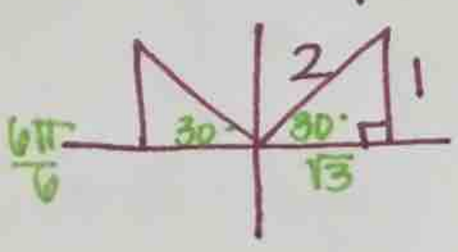
OR where x=-1 on unitcircle

3. $\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \theta$ $\frac{\pi}{3}$ ref ∠



$\boxed{\frac{\pi}{3}, \frac{4\pi}{3}}$

4. $\csc^{-1}\left(\frac{2}{1}\right) = \theta$ $\frac{HYP}{OPP}$ $\frac{\pi}{6}$ ref ∠

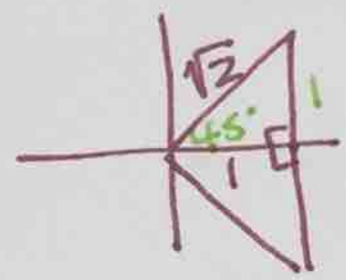


$\boxed{\frac{\pi}{6}, \frac{5\pi}{6}}$

5. $\arccos\left(\frac{\sqrt{2}}{2}\right) = \theta$

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\frac{\pi}{4}$ ref ∠

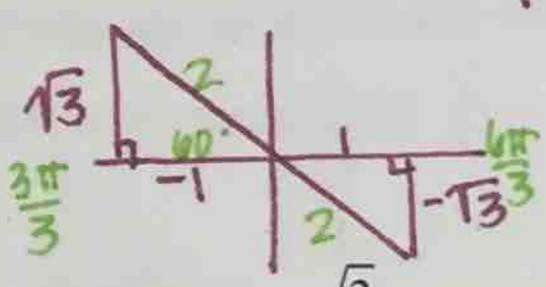


$\boxed{\frac{\pi}{4}, \frac{7\pi}{4}}$

Find the general solutions to the equation

6. $\tan x = -\sqrt{3}$ $+2\pi n$

$x = \tan^{-1}(-\sqrt{3})$ ref ∠ $\frac{\pi}{3}$

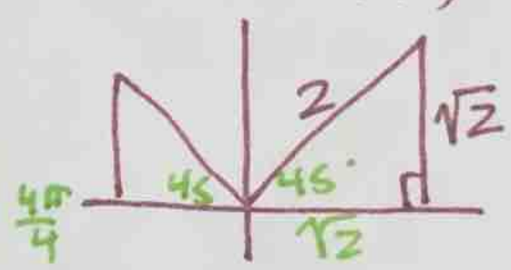


$\boxed{\frac{2\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n}$ OR $\frac{2\pi}{3} + \pi n$

7. $\sin x = \frac{\sqrt{2}}{2}$ ref ∠ $\frac{\pi}{4}$

$x = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{\pi}{4} + 2\pi n, \frac{3\pi}{4} + 2\pi n}$



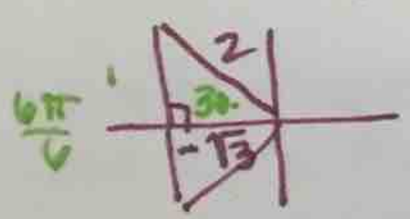
$(\sqrt{2})^2 + x^2 = 2^2$

$x = \sqrt{2}$

same legs → 45/45/90°

8. $\cos x = \frac{-\sqrt{3}}{2}$

$x = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{\pi}{6}$ ref ∠



$\boxed{\frac{5\pi}{6} + 2\pi n, \frac{7\pi}{6} + 2\pi n}$

9. $\csc x = 1$

$x = \csc^{-1}(1)$ where does $\frac{1}{\sin} = 1$?

NO △ ∴

$\csc = \frac{1}{\sin}$

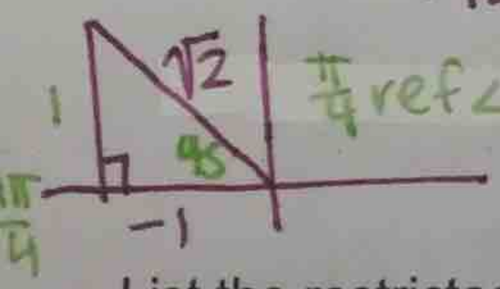
	S	CT
0	0	1 0
$\pi/2$	1	0 U
π	0	-1 0
$3\pi/2$	-1	0 U

$\boxed{\frac{\pi}{2} + 2\pi n}$

Find the exact principal value (one answer)

10. $\cos x = \frac{-1}{\sqrt{2}}$ QI & II (+ / −)

$x = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$



$\frac{\pi}{4}$ ref ∠

$\boxed{\frac{3\pi}{4}}$

11. $\sin x = -1$

no △

$x = \sin^{-1}(-1)$ $y = -1$

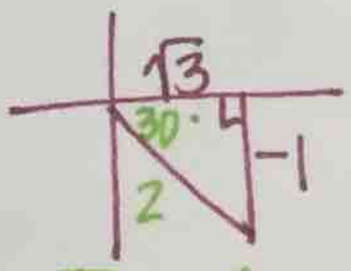
	S	CT
0	0	1 0
$\pi/2$	1	0 U
π	0	-1 0
$3\pi/2$	-1	0 U

$\frac{3\pi}{2}$ is not in RR

$\boxed{\frac{-\pi}{2}}$

12. $\tan x = \frac{-\sqrt{3}}{3}$ QI & IV (+ / −)

$x = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$



$\frac{\pi}{6}$ ref ∠

$\boxed{\frac{-\pi}{6}}$

List the restricted ranges of each

13. $y = \sin^{-1} x$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

14. $y = \cos^{-1} x$

$[0, \pi]$

15. $y = \tan^{-1} x$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Find the exact value using radicals or radians when necessary

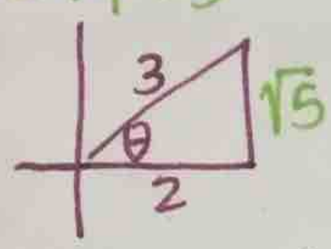
$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

	S	C	T
0	0	1	0
$\pi/2$	1	0	U
π	0	−1	0
$3\pi/2$	−1	0	U

14. $\sin\left(\cos^{-1}\left(\frac{2}{3}\right)\right)$

$2^2+y^2=3^2$

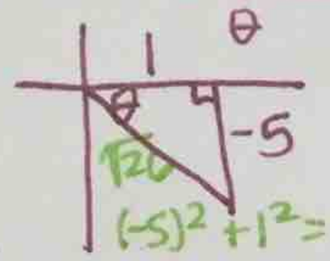
θ (angle)



$\sin\theta = \frac{O}{H} = \boxed{\frac{\sqrt{5}}{3}}$

15 $\sin\left(\tan^{-1}\left(\frac{-5}{1}\right)\right)$

θ



$(-5)^2+1^2=c^2$

$\sin\theta = \boxed{\frac{-5}{\sqrt{26}}}$

16. $\sin^{-1}(\cos(0))$

RATIO

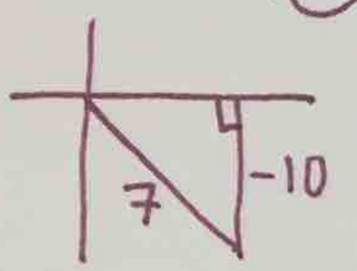
$\cos 0 = 1$

$\sin^{-1}(1) = \boxed{\frac{\pi}{2}}$

Angle

17. $\cos\left(\arcsin\left(-\frac{10}{7}\right)\right)$

angle

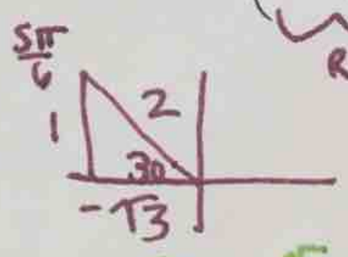


△ not possible

$\boxed{\text{DNE}}$

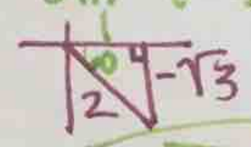
18. $\sin^{-1}\left(\cos\left(\frac{5\pi}{6}\right)\right)$

ratio



$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

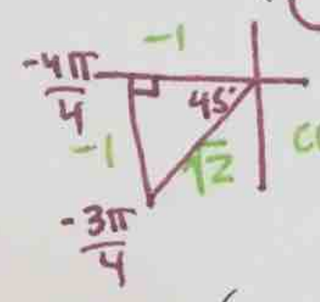
$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$



$\boxed{-\frac{\pi}{3}}$

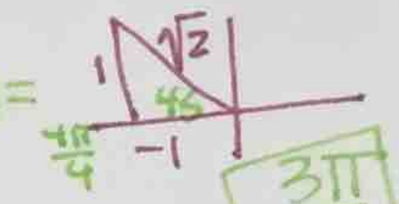
19. $\cos^{-1}\left(\cos\left(-\frac{3\pi}{4}\right)\right)$

ratio



$\cos\left(-\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$



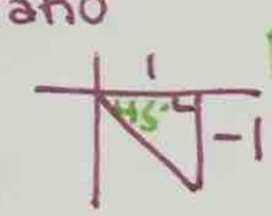
$\boxed{\frac{3\pi}{4}}$

20. $\tan^{-1}(\cos(\pi))$

ratio

$\cos\pi = -1$

$\tan^{-1}\left(\frac{-1}{1}\right)$



$\boxed{\frac{-\pi}{4}}$

21. $\tan(\cos^{-1}(0))$

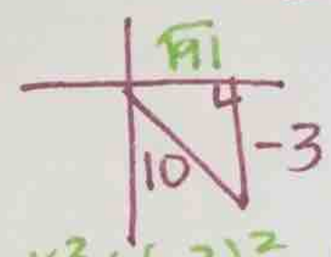
angle

$\cos^{-1}(0) = \frac{\pi}{2}$

$\tan\left(\frac{\pi}{2}\right) = \boxed{U}$

22. $\tan\left(\text{arc}\csc\left(-\frac{10}{3}\right)\right)$ $\frac{H}{O}$

angle



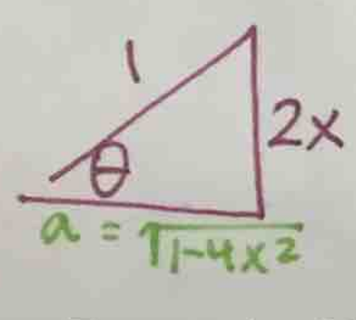
$x^2+(-3)^2=10^2$

$\tan\theta = \boxed{\frac{-3}{\sqrt{91}}}$

Write as an algebraic expression

23. $\tan(\arcsin(2x))$ $\frac{O}{H}$

$\frac{2x}{1}$



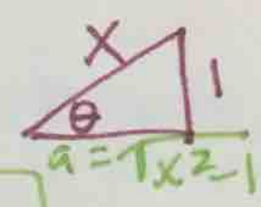
$(2x)^2+a^2=1^2$

$a=\sqrt{1-4x^2}$

$\tan\theta = \frac{O}{A} = \boxed{\frac{2x}{\sqrt{1-4x^2}}}$

24. $\cos(\csc^{-1}x)$ $\frac{H}{O}$

$\frac{x}{1}$

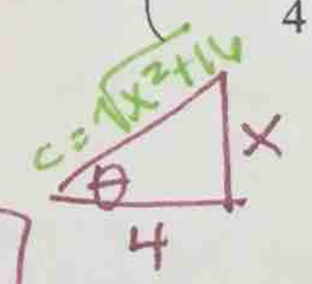


$a^2+1^2=x^2$

$a=\sqrt{x^2-1}$

$\cos\theta = \frac{A}{H} = \boxed{\frac{\sqrt{x^2-1}}{x}}$

25. $\sin\left(\tan^{-1}\frac{x}{4}\right)$ $\frac{O}{A}$



$x^2+4^2=c^2$

$\sqrt{x^2+16}=c$

$\sin\theta = \frac{O}{H} = \boxed{\frac{x}{\sqrt{x^2+16}}}$

For each of the following functions use **algebraic** methods to find the general solutions for the given value of $f(x)$ Round to three decimal places.

26. $y=5+4\sin\frac{\pi}{12}(x+10)$

$f(x)=7$

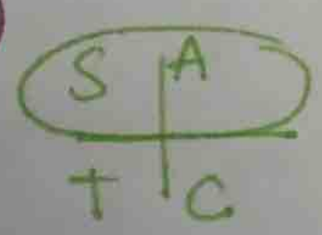
$7=5+4\sin\frac{\pi}{12}(x+10)$

$-5\quad -5$

$\frac{2}{4}=\frac{4\sin\frac{\pi}{12}(x+10)}{4}$

$\frac{1}{2}=\sin\frac{\pi}{12}(x+10)$

$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{12}(x+10)$



QI $\frac{12}{\pi}(0.524+2\pi n)=\left(\frac{\pi}{12}(x+10)\right)\frac{12}{\pi}$

$2+24n=x+10$

$-10\quad -10$

$\boxed{-8+24n=x}$

QII. $(\pi-.524)+2\pi n=\frac{\pi}{12}(x+10)$

$\frac{12}{\pi}(2.618+2\pi n)=\left(\frac{\pi}{12}(x+10)\right)\frac{12}{\pi}$

$10+24n=x+10$

$-10\quad -10$

$\boxed{0+24n=x}$

AND

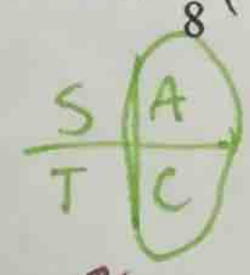
27. $y=1+3\cos\frac{\pi}{8}(x+7)$

$f(x)=2$

$2=1+3\cos\frac{\pi}{8}(x+7)$

$\frac{1}{3}=\cos\frac{\pi}{8}(x+7)$

$\cos^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{8}(x+7)$



QI $\frac{8}{\pi}(1.231+2\pi n)=\left(\frac{\pi}{8}(x+7)\right)\frac{8}{\pi}$

$3.135+16n=x+7$

$\boxed{-3.865+16n=x}$

QIV. $-1.231+2\pi n=\frac{\pi}{8}(x+7)$

$-3.135+16n=x+7$

$\boxed{-10.135+16n=x}$

QIII. −ref∠