

Name: _____

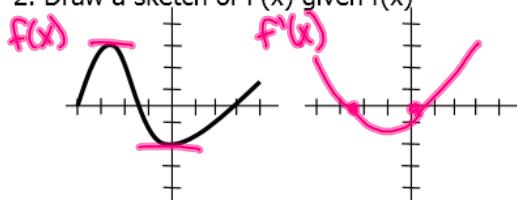
Derivative Review

1. What's the difference between
- average rate of change
- and
- instantaneous rate of change
- ?

Secant line
(find slope between 2 pts)

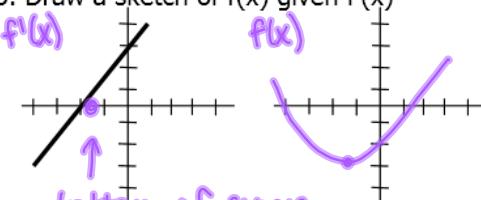
Tangent line
(find derivative)

2. Draw a sketch of
- $f'(x)$
- given
- $f(x)$



tops & bottoms of curves on $f(x)$
mean x-intercepts on $f'(x)$

3. Draw a sketch of
- $f(x)$
- given
- $f'(x)$



bottom of curve
(goes from neg. slope to pos.)

4. The position of an object is given by
- $s = 3t^2 - 4t + 6$
- . What is the
- average velocity
- over the interval
- $[1, 4]$
- ?

$$\frac{\Delta s}{\Delta t} = \frac{s(4) - s(1)}{4 - 1} = \frac{(3(4)^2 - 4(4) + 6) - (3(1)^2 - 4(1) + 6)}{4 - 1} = \frac{33}{3} = 11$$

5. Given the position of a function
- $F(x) = 2x^3 - 3x^2 + 7$
- , what is the
- instantaneous rate of change
- of
- F
- ?

$$F'(x) = 6x^2 - 6x$$

6. Given the position of a function
- $s = t^4 - 2t + 3$
- , what is the instantaneous rate of change at
- $t=2$
- ?

$$s' = 4t^3 - 2$$

$$s'(2) = 4(2)^3 - 2 = 30$$

derivative when $t=2$

For 7-8, use the formal (limit) definition of the derivative to find the derivative

$$7. f(x) = \sqrt{3x-1}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{3(x+h)-1} - \sqrt{3x-1}}{h} \cdot \frac{\sqrt{3(x+h)-1} + \sqrt{3x-1}}{\sqrt{3(x+h)-1} + \sqrt{3x-1}}$$

multiply by conjugate

$$\lim_{h \rightarrow 0} \frac{(3x+3h-1) - (3x-1)}{h(\sqrt{3x+3h-1} + \sqrt{3x-1})} = \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{3x+3h-1} + \sqrt{3x-1})} = \lim_{h \rightarrow 0} \frac{3}{\sqrt{3x+3h-1} + \sqrt{3x-1}}$$

$$9. \text{ Find } \lim_{h \rightarrow 0} \frac{3(x+h)^3 - 3x^3}{h}$$

$$= \boxed{\frac{3}{2\sqrt{3x-1}}}$$

$$8. f(x) = 3x^2 - 4x$$

$$\lim_{h \rightarrow 0} \frac{(3(x+h)^2 - 4(x+h)) - (3x^2 - 4x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 4x - 4h - 3x^2 + 4x}{h}$$

$$\lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 4x - 4h - 3x^2 + 4x}{h}$$

$$\lim_{h \rightarrow 0} \frac{6xh + 3h^2 - 4h}{h}$$

$$11. \text{ Find } \lim_{h \rightarrow 0} \frac{\sqrt{16+h} + \sqrt{16}}{h}$$

$$= \boxed{0x-4}$$

change eqns (FOIL)
dis

$$f(x) = 3x^3$$

$$f'(x) = 9x^2$$

$$f(x) = \frac{3}{x} = 3x^{-1}$$

$$f'(x) = -3x^{-2} = \boxed{\frac{-3}{x^3}}$$

$$f(x) = \sqrt{x} = x^{1/2} \text{ when } x=16$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{2\cdot 4} =$$

* #9-11 are all the limit definition of the derivative
If you can recognize the function, just use power rule!!

$$f(x) = 3x^{-1} - 8x + 1$$

12. Find $f'(x)$ for $f(x) = \frac{3}{x} - 8x + 1$

$$f'(x) = -3x^{-2} - 8 = \boxed{\frac{-3}{x^2} - 8}$$

14. For $f(x) = x^4 + 3x^2 - 2$ find $f'(x)$, $f''(x)$, $f'''(x)$ and $f^{IV}(x)$

$$f'(x) = 4x^3 + 6x$$

$$f''(x) = 12x^2 + 6$$

$$f'''(x) = 24x$$

$$f^{IV}(x) = 24$$

15. Find the equation of the tangent line to $f(x) = 2x(x-3)$ at $x = 2$

$$y+4 = 2(x-2)$$

* know how to manipulate into other forms

16. Find the equation of the tangent line to $f(x) = \sqrt{x-2}$ at $x = 6$

$$y-2 = \frac{1}{4}(x-6)$$

$$f(2) = 2(2)^2 - 6(2) = -4$$

so point = $(2, -4)$

$$f(x) = 2x^2 - 6x$$

$$f'(x) = 4x - 6 \rightarrow f'(2) = 4(2) - 6 = 2$$

$\uparrow m$ for the tangent line

$$f(x) = (x-2)^{1/2}$$

$$f'(x) = \frac{1}{2}(x-2)^{-1/2} \rightarrow f'(6) = \frac{1}{2\sqrt{6-2}} = \frac{1}{4}$$

17. Knowing that $f(-3) = 12$, $f'(-3) = 9$, $g(-3) = -4$, $g'(-3) = 7$, $h(-3) = -2$ and $h'(-3) = 5$, determine

A. $(fg)'(-3)$ (product)	B. $(\frac{h}{g})'(-3)$ (quotient)	C. $(\frac{fg}{h})'(-3)$ $h(-3) fg'(-3) - fg(-3) h'(-3)$
$f(-3) g'(-3) + g(-3) f'(-3)$	$\frac{g(-3) h'(-3) - h(-3) g'(-3)}{(g(-3))^2}$	$\frac{(h(-3))^2}{(h(-3))^2}$
$(12)(7) + (-4)(9)$	$\frac{(-4)(5) - (-2)(7)}{(7)^2}$	$\frac{(5)(48) - (12)(-4)(5)}{(-2)^2}$
$\boxed{48}$	$\boxed{\frac{-6}{49}}$	$\frac{240 + 240}{4} = \boxed{120}$

Find the derivative of the function

$$18. f(x) = (2 - \sqrt{x})(3x - 2x^3)$$

$$f'(x) = (2 - \sqrt{x})(3 - 6x^2) + (-\frac{1}{2\sqrt{x}})(3x - 2x^3)$$

* will be easier to simplify on test!

$$19. f(x) = \frac{4x^2 \sin x}{8x \cos x}$$

$$f'(x) = 8x \sin x + 4x^2 \cos x$$

20. $f(x) = \underbrace{(1+\sqrt[3]{x^3})}_{\frac{3x^{1/2}}{2}} \underbrace{(\frac{1}{x^3} - 2\sqrt[3]{x})}_{-3x^{-4}} \quad \text{PRODUCT RULE}$

$$\frac{3x^{1/2}}{2} - 3x^{-4} - \frac{2}{3x^{2/3}}$$

$$f'(x) = \left(\frac{3\sqrt{x}}{2} \right) \left(\frac{1}{x^3} - 2\sqrt[3]{x} \right) + (1+\sqrt[3]{x^3}) \left(-\frac{3}{x^4} - \frac{2}{3x^{2/3}} \right) \quad \text{ew}$$

21. $g(y) = \frac{y^2 - 1}{y^2 + 1} \quad \text{QUOTIENT RULE}$

$\frac{d\text{high}}{dy} = 2y$
 $\frac{d\text{low}}{dy} = 2y$

$$g'(y) = \frac{(y^2 + 1)(2y) - (y^2 - 1)(2y)}{(y^2 + 1)^2}$$

22. $h(z) = \frac{(1-4z)(2+z)}{3+9z} = \frac{2-9z-4z^2}{3+9z} \quad \frac{d\text{high}}{dz} = -9-8z$
 $\frac{d\text{low}}{dz} = 9$

$$h'(z) = \frac{(3+9z)(-9-8z) - (2-9z-4z^2)(9)}{(3+9z)^2}$$

23. $h(x) = \frac{2x^3}{\cos x} \quad \text{QUOTIENT RULE}$

$\frac{d\text{high}}{dx} = 6x^2$
 $\frac{d\text{low}}{dx} = -\sin x$

$$h'(x) = \frac{\cos x(6x^2) - 2x^3(-\sin x)}{(\cos x)^2} = \boxed{\frac{6x^2 \cos x + 2x^3 \sin x}{\cos^2 x}}$$

24. $f(x) = \frac{2x-\sqrt{x}}{6} \quad \text{not an actual quotient}$

$$f'(x) = \frac{1}{6} \left(2 - \frac{1}{2\sqrt{x}} \right) \quad \text{OR} \quad \frac{1}{3} - \frac{1}{12\sqrt{x}}$$

25. $f(x) = x \left(\frac{2}{x^3} - \frac{3x}{x-1} \right) = \frac{\frac{2}{x^2}}{\frac{2x^{-2}}{x^2}} - \frac{\frac{3x^2}{x-1}}{\frac{x-1}{x-1}} \quad \frac{d\text{high}}{dx} = 6x$
 $\frac{d\text{low}}{dx} = 1$

$$f'(x) = \frac{-4}{x^3} - \frac{(x-1)(6x) - 3x^2(1)}{(x-1)^2} \quad \text{this makes me tired}$$

Find $f'(1)$ for each function

26. $f(x) = \underbrace{(x^2 - 5x + 1)}_{2x-5} \underbrace{(12 + 2x - x^3)}_{2-3x^2}$ PRODUCT RULE

$$f'(x) = (2x-5)(12+2x-x^3) + (2-3x^2)(x^2-5x+1)$$

$$f'(1) = (2(1)-5)(12+2(1)-(1)^3) + (2-3(1)^2)(1^2-5(1)+1)$$

$$(2)(13) + (-3)(-4) = \boxed{-36}$$

27. $f(x) = \frac{\sqrt[3]{x}}{1+x^2}$
 dhigh = $\frac{1}{3x^{2/3}}$
 dlow = $2x$

$$F'(x) = \frac{(1+x^2) \left(\frac{1}{3x^{2/3}} \right) - x^{1/3} (2x)}{(1+x^2)^2}$$

$$f'(1) = \frac{(1+1^2) \left(\frac{1}{3(1^{2/3})} \right) - (1)^{1/3} (2(1))}{(1+1^2)^2} = \frac{\frac{2}{3} - 2}{2^2} = \frac{-\frac{4}{3}}{4} = \boxed{-\frac{1}{3}}$$

- * memorize limit definition, power, product, & quotient rules
- * study worksheets → test questions might repeat!
- * you got this ☺