10.2 Half Angle Properties

Name: _____

Find the exact value of $\sin\frac{1}{2}x$**,** $\cos\frac{1}{2}x$**, and** $\tan\frac{1}{2}x$ **under the given conditions:**

1. $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$

2. $\cos x = -\frac{3}{5}$ and $\frac{\pi}{2} < x < \pi$

Using the half angle properties, prove the following.

3. $\tan\frac{1}{2}x + \cot\frac{1}{2}x = 2\csc x$

4. $\tan x \tan\frac{1}{2}x = \sec x - 1$

5. $\dfrac{2\tan\frac{1}{2}x}{1+\tan^2\frac{1}{2}x} = \sin x$

6. $\tan\frac{1}{2}x(2\cot x + \tan\frac{1}{2}x) = 1$

7. $\tan\frac{1}{2}x = \csc x - \cot x$

8. $\dfrac{\cos\frac{1}{2}x - \sin\frac{1}{2}x}{\cos\frac{1}{2}x + \sin\frac{1}{2}x} = \dfrac{\cos x}{1+\sin x}$

9. $\dfrac{\cos\frac{1}{2}x + \sin\frac{1}{2}x}{\cos\frac{1}{2}x - \sin\frac{1}{2}x} = \sec x + \tan x$

10. $\tan(\frac{\pi}{4} + \frac{x}{2}) = \sec x + \tan x$