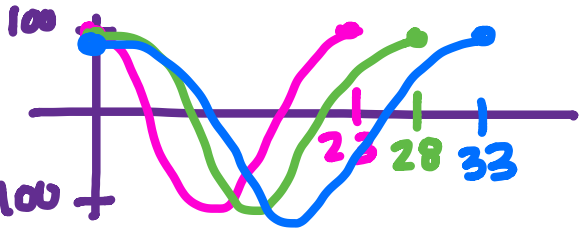
HW 3.4 – Sinusoidal Functions as Mathematical Models (3)

1. According to biorhythm theory, your body is governed by three independent sinusoidal functions, each with a different period as follows:

- Physical function: Period = 23 days
- Emotional function: Period = 28 days
- Intellectual function: Period = 33 days



D=0

a. You are at a high point on all three cycles today! This means that you are at your very highest ability physically, emotionally, and intellectually. The amplitude of each sinusoid is 100 units and the vertical displacement of each is zero. Write equations for these three functions in terms of the number of days after today.

A=100

C=0

today ⇒ x=0

Physical: B: $\frac{2\pi}{23}$

$y = 100\cos\frac{2\pi}{23}x$

Emotional B: $\frac{2\pi}{28} = \frac{\pi}{14}$

$y = 100\cos\frac{\pi}{14}x$

Intellectual B: $\frac{2\pi}{33}$

$y = 100\cos\frac{2\pi}{33}x$

b. 33 days from now you will again be at an intellectual high point. What will be the values of your physical and emotional functions on that day?

x = 33

-91.72 (physical)

x = 33

43.39 (emotional)

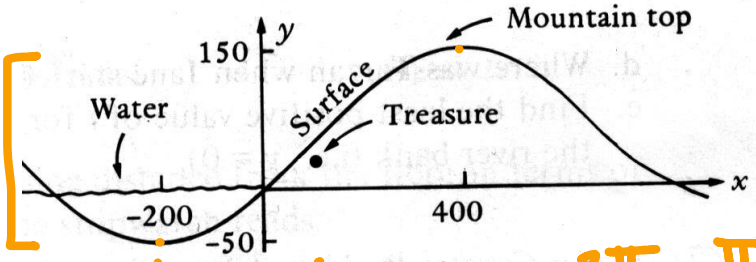
c. Biorhythm theory says that the most dangerous time for a particular function is when it crosses the sinusoidal axis. When is the first time this will happen for each of your physical, emotional, and intellectual functions? y = 0

P: 5.75

E: 7

I: 8.25

2. You are on an expedition seeking a treasure that is buried in the side of a mountain. The mountain range has a sinusoidal cross-section. A valley to the left is filled with water to a depth of 50 meters, and the top of the mountain is 150 meters above the water level. You set up an x-axis at water level and a y-axis 200 meters to the right of the deepest part of the water. The top of the mountain is at $x = 400$ meters.



200

C = 150 − 100 = 50

Amp: 100

$\frac{1}{2}$ per = 600 ⇒ $B = \frac{2\pi}{1200} = \frac{\pi}{600}$

D: cos 400

a. Write an equation expressing y in terms of x for points on the surface of the mountain.

$y = 50 + 100\cos\frac{\pi}{600}(x - 400)$

b. Does this graph pass through the origin? Prove your answer algebraically.

(0, 0) (x, y)

$0 \stackrel{?}{=} 50 + 100\cos\frac{\pi}{600}(0 - 400)$

$0 \stackrel{?}{=} 50 + 100(-0.5)$

$0 = 50 - 50$ ✓

Yes

c. The treasure is located within the mountain at the point $(130, 40)$. (This point is NOT on the graph!) Would it be shorter to dig a horizontal or vertical tunnel to the treasure? Justify your answer.

(x, y)

$y = 50 + 100\cos\frac{\pi}{600}(130 - 400)$

$y = 65.043$

$40 = 50 + 100\cos\frac{\pi}{600}(x - 400)$

$x = 80.869$

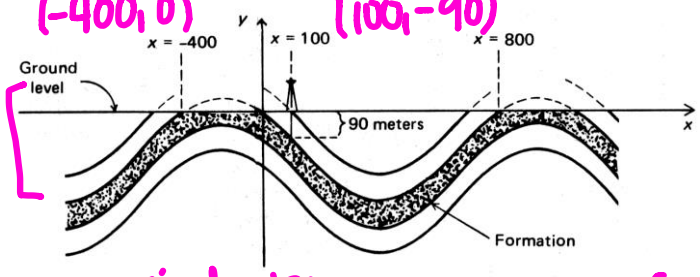
vertically: 65 − 40 = 25 m

horizontally: 130 − 81 = 49 m

vertical tunnel

3. An old rock formation is warped into the shape of a sinusoid. Over the centuries, the top has eroded away, leaving the ground with a flat surface from which various layers of rock are cropping out. Since you have studied sinusoids, the geologists call upon you to predict the depth of a particular formation at various points. You construct an x-axis along the ground and a y-axis at the edge of an outcropping as shown. A hole drilled at $x = 100$ meters shows that the top of the formation is 90 meters deep at that point.

(-400, 0) (100, -90)



a. Write an equation of the formation expressing y in terms of x.

$y = -90 + 180 \cos\frac{\pi}{600}(x + 200)$?

(see work on next page)

period = 1200

$B = 2\pi/1200 = \pi/600$

D: cos -200

b. If a hole were drilled to the top of the formation at $x = 510$, how deep would it be?

x = 510

-240.961 m

c. What is the maximum depth of the top of the formation, and what is the first positive x-value where it reaches this depth?

-270m (lowest y-value)

x = 400 m

d. How high above the present ground level did the formation reach before it eroded away?

90 m (highest y-value)

e. For what values x between 0 and 800 meters is the top of the formation within 120 meters of the surface? y = -120

(0, 131.98) and (668.020, 800)

131.980 m & 668.020 m

f. The geologists decide to drill holes to the top of the formation every 150 meters from $x = 50$ through $x = 800$. The drilling costs $75 per meter of depth. Find the cost of drilling each hole and the total cost of the drilling. cost = depth x 75

x	y	cost
50	-43.41	$3256
200	-180	$13500
350	-263.9	$19790
500	-245.9	$18441
650	-136.6	$10244
800	0	$0

Total cost:

$65,175

$y = C + A\cos\frac{\pi}{600}(x+200)$

use a system of equations & the points you know

$$\begin{cases} 0 = C + A\cos\frac{\pi}{600}(-400+200) \\ -90 = C + A\cos\frac{\pi}{600}(100+200) \end{cases}$$

$0 = C + A\cos\frac{\pi}{600}(-200)$

$0 = C + 0.5A$

$-90 = C + A\cos\frac{\pi}{600}(300)$

$-90 = C + A(0)$

$-90 = C$

$0 = -90 + 0.5A$

$90 = 0.5A$

$180 = A$