5.1 Reciprocal, Quotient, and Pythagorean Identities

2. cscx tanx to secx

$$\frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cos x}$$

$$\frac{1}{\cos x}$$

$$\sec x$$

4. cscx tanx cosx to 1

$$\frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \cancel{\cos x}$$

$$1$$

6. $\cos^2 x$ cscx secx to cotx

$$\cos^2 x \left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)$$

$$\frac{\cos^{\cancel{2}} x}{\sin x \cancel{\cos x}}$$

$$\frac{\cos x}{\sin x}$$

$$\cot x$$

8. sinx + cotx cosx to cscx

$$\sin x + \frac{\cos x}{\sin x} \cdot \cos x$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$\frac{1}{\sin x}$$

$$\csc x$$

10. secx - cosx to sinx tanx

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$

$$\frac{1-\cos^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x}$$

$$\frac{\sin x}{\cos x} \cdot \sin x$$

$$\tan x \sin x$$

12. cosx(secx + cosx csc²x) to csc²x

$$\cos x\left(\frac{1}{\cos x} + \frac{\cos x}{\sin^2 x}\right)$$

$$\frac{\cancel{\cos x}}{\cancel{\cos x}} + \frac{\cos^2 x}{\sin^2 x}$$

$$1 + \cot^2 x$$

$$\csc^2 x$$

14. (secx - 1)(secx + 1) to tan²x

$$\sec^2 x - 1$$

$$\tan^2 x$$

16. $(1 - \tan x)^2$ to $\sec^2 x - 2\tan x$

$$1 - 2\tan x + \tan^2 x$$

$$\underbrace{1 + \tan^2 x} - 2\tan x$$

$$\sec^2 x - 2\tan x$$

18. $(\cos x - \sec x)^2$ to $\tan^2 x - \sin^2 x$

$$\cos^2 x - 2\cos x \sec x + \sec^2 x$$

$$(\cancel{1} - \sin^2 x) - \cancel{2(1)} + (\tan^2 x \cancel{+1})$$

$$\tan^2 x - \sin^2 x$$

20. $\dfrac{1 - \cos^2 x}{\tan x}$ to sin x cos x

$\dfrac{\sin^2 x}{\tan x}$

$\dfrac{\sin^2 x}{\dfrac{\sin x}{\cos x}}$

$\sin^{\cancel{2}} x \cdot \dfrac{\cos x}{\cancel{\sin x}}$

$\sin x \cos x$ ☺

22. $\dfrac{1 + \cot^2 x}{\sec^2 x}$ to $\cot^2 x$

$\dfrac{\csc^2 x}{\sec^2 x}$

$\cot^2 x$ ☺

24. $\dfrac{\csc x}{\cos x} - \dfrac{\cos x}{\sin x}$ to tan x

$\dfrac{\csc x \sin x - \cos^2 x}{\cos x \sin x}$

$\dfrac{1 - \cos^2 x}{\cos x \sin x}$

$\dfrac{\sin^{\cancel{2}} x}{\cos x \cancel{\sin x}}$

$\dfrac{\sin x}{\cos x}$

$\tan x$ ☺

26. $\dfrac{1}{\sec x - \tan x} + \dfrac{1}{\sec x + \tan x}$ to $2\sec x$

$$\frac{\sec x + \cancel{\tan x} + \sec x - \cancel{\tan x}}{(\sec x - \tan x)(\sec x + \tan x)}$$

$$\frac{2\sec x}{\sec^2 x - \tan^2 x}$$

$$\frac{2\sec x}{1}$$

$1 + \tan^2 x = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$