② $\tan x(\cot x + \tan x) = \sec^2 x$

$\tan x \cot x + \tan^2 x$

$\tan x \cdot \frac{1}{\tan x} + \tan^2 x$

$1 + \tan^2 x$

$\cancel{1} + \sec^2 x - \cancel{1}$

$\sec^2 x$ ☺

④ $\cos x(\sec x - \cos x) = \sin^2 x$

$\cos x \sec x - \cos^2 x$

$1 - \cos^2 x$

$\sin^2 x$ ☺

⑥ $\cos^2 x + \tan^2 x \cos^2 x = 1$

$\cos^2 x(1 + \tan^2 x)$

$\cos^2 x(\sec^2 x)$

$\cos^2 x\left(\frac{1}{\cos^2 x}\right)$

1 ☺

⑧ $(1+\sin x)(1-\sin x) = \cos^2 x$

$1 - \sin x + \sin x - \sin^2 x$

$1 - \sin^2 x$

$\cos^2 x$ ☺

⑩ $\cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$

$\cot^2 x(\csc^2 x - 1)$

$\cot^2 x(\cot^2 x)$

$\cot^4 x$ ☺

⑫ $\sec^4 x - \tan^4 x = 1 + 2\tan^2 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$1(1 + \tan^2 x + \tan^2 x)$

$1 + 2\tan^2 x$ ☺

(14) $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$

$\frac{\cos x \sec x - \sin^2 x}{\sin x \cos x}$

$\frac{\cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} - \sin^2 x}{\sin x \cos x}$

$\frac{1 - \sin^2 x}{\sin x \cos x}$

$\frac{\cos^{\cancel{2}} x}{\sin x \cancel{\cos x}}$

$\frac{\cos x}{\sin x}$

$\cot x$ ☺

(16) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$

$\cos^2 x + \sin^2 x$ Pythag.

1 ☺

(18) $\frac{1}{1-\sin x} = \sec^2 x + \sec x \tan x$

$\frac{1+\sin x}{1+\sin x} \cdot \frac{1}{1-\sin x}$

$\frac{1+\sin x}{1-\sin^2 x}$

$\frac{1+\sin x}{\cos^2 x}$

$\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$

$\sec^2 x + \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$\sec^2 x + \tan x \sec x$ ☺

(20) $\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x} = 2\csc x$

$$\frac{\sin^2 x + (1-\cos x)^2}{\sin x(1-\cos x)}$$

$$\frac{\sin^2 x + 1 - 2\cos x + \cos^2 x}{\sin x(1-\cos x)}$$ Pythag

$$\frac{1+1-2\cos x}{\sin x(1-\cos x)}$$

$$\frac{2-2\cos x}{\sin x(1-\cos x)}$$

$$\frac{2\cancel{(1-\cos x)}}{\sin x\cancel{(1-\cos x)}}$$

$$2 \cdot \frac{1}{\sin x}$$

$$2\csc x$$ ☺

(22) $\frac{1+\sin x}{1-\sin x} = 2\sec^2 x + 2\sec x \tan x - 1$

$$\frac{1+\sin x}{1+\sin x}\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$\frac{(1+\sin x)(1+\sin x)}{1-\sin^2 x}$$

$$\frac{1+2\sin x+\sin^2 x}{\cos^2 x}$$

$$\frac{1+2\sin x+(1-\cos^2 x)}{\cos^2 x}$$

$$\frac{2+2\sin x-\cos^2 x}{\cos^2 x}$$

$$\frac{2}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$$

$$2\sec^2 x + \frac{2}{\cos x}\cdot\frac{\sin x}{\cos x} - 1$$

$$2\sec^2 x + 2\sec x \tan x - 1$$ ☺ (Phew!)

(24) $\sin^3 x \cos^2 x = \cos^2 x \sin x - \cos^4 x \sin x$

$\cos^2 x \sin x (1 - \cos^2 x)$

$\cos^2 x \sin x (\sin^2 x)$

$\cos^2 x \sin^3 x$ ☺

(26) $\sec x + \tan x = \dfrac{1}{\sec x - \tan x}$

$\dfrac{1}{\sec x - \tan x} \cdot \dfrac{\sec x + \tan x}{\sec x + \tan x}$

$\dfrac{\sec x + \tan x}{\sec^2 x - \tan^2 x}$ pythag

$\dfrac{\sec x + \tan x}{1}$ ☺

(28) $\dfrac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5} = \dfrac{\tan x - 4}{\tan x + 2}$

$1 + \tan^2 x = \sec^2 x$

$\dfrac{(1 + \tan^2 x) - 6\tan x + 7}{(1 + \tan^2 x) - 5}$

$\dfrac{\tan^2 x - 6\tan x + 8}{\tan^2 x - 4}$

$\dfrac{(\tan x - 4)\cancel{(\tan x - 2)}}{\cancel{(\tan x - 2)}(\tan x + 2)}$

$\dfrac{\tan x - 4}{\tan x + 2}$ ☺