

4.5 Sum and Difference Properties (3)

Name: _____

Show that the trig functions do NOT distribute over addition and subtraction by letting $A = 60^\circ$ and $B = 90^\circ$:

1. $\sin(A+B) \neq \sin A + \sin B$

$$\sin 150^\circ \quad \sin 60^\circ + \sin 90^\circ$$

$$\frac{1}{\sqrt{3}} \neq \frac{\sqrt{3}}{2} + 1$$

2. $\cot(A-B) \neq \cot A - \cot B$

$$\cot(-30^\circ) \quad \cot 60^\circ - \cot 90^\circ$$

$$-\sqrt{3} \neq \frac{1}{\sqrt{3}} - 0$$

3. $\sec(A+B) \neq \sec A + \sec B$

$$\frac{-2}{\sqrt{3}} \neq 2 + \text{und.}$$

4. $\csc(A-B) \neq \csc A - \csc B$

$$-2 \neq \frac{2}{\sqrt{3}} - 1$$

Demonstrate that the given property is true by substituting $A = \frac{2\pi}{3}$ and $B = \frac{\pi}{6}$:

5. $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

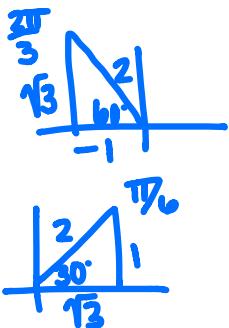
$$\cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \cos\frac{2\pi}{3} \cdot \cos\frac{\pi}{6} + \sin\frac{2\pi}{3} \cdot \sin\frac{\pi}{6}$$

$$\cos\frac{\pi}{2} = \left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$0 = -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$0 = 0 \checkmark$$

	S	C	I
0	0	1	0
$\pi/2$	1	0	V
π	0	-1	0
$3\pi/2$	-1	0	U



6. $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$$1 = 1 \checkmark$$

Prove that the given equation is an identity:

7. $\cos\left(\theta - 90^\circ\right) = \sin \theta$

$$\frac{\cos \theta \cos 90^\circ + \sin \theta \sin 90^\circ}{\cos \theta (0) + \sin \theta (1)}$$

$$\frac{\sin \theta}{\sin \theta} \quad \text{∴}$$

9. $\tan\left(x - 90^\circ\right) = -\cot x$

$$\frac{\sin(x - 90)}{\cos(x - 90)}$$

$$\frac{\sin x \cos 90 - \cos x \sin 90}{\cos x \cos 90 + \sin x \sin 90}$$

$$\frac{\sin x (0) - \cos x (1)}{\cos x (0) + \sin x (1)}$$

$$\frac{-\cos x}{\sin x} \Rightarrow -\cot x \quad \text{∴}$$

11. $\sin(x + \pi) = -\sin x$

$$\frac{\sin x \cos \pi + \cos x \sin \pi}{\sin x (-1) + \cos x (0)}$$

$$-\sin x \quad \text{∴}$$

8. $\sec\left(x - \frac{\pi}{2}\right) = \csc x$

$$\frac{1}{\cos(x - \frac{\pi}{2})}$$

$$\frac{1}{\cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2}}$$

$$\frac{1}{\cos x (0) + \sin x (1)} \Rightarrow \frac{1}{\sin x} \Rightarrow \csc x \quad \text{∴}$$

10. $\cos\left(x - \frac{3\pi}{2}\right) = -\sin x$

$$\frac{\cos x \cos \frac{3\pi}{2} + \sin x \sin \frac{3\pi}{2}}{\cos x (0) + \sin x (-1)}$$

$$-\sin x \quad \text{∴}$$

12. $\frac{\cos(x - y)}{\sin x \cos y} = \cot x + \tan y$

$$\frac{\cos x \cos y + \sin x \sin y}{\sin x \cos y}$$

$$\frac{\cancel{\cos x \cos y}}{\cancel{\sin x \cos y}} + \frac{\cancel{\sin x \sin y}}{\cancel{\sin x \cos y}}$$

$$\cot x + \tan y \quad \text{∴}$$

13. $\sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{3}\right) = \cos x$

$$\left(\sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6}\right) + \left(\cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3}\right)$$

$$\underline{\sin x \left(\frac{\sqrt{3}}{2}\right)} + \underline{\cos x \left(\frac{1}{2}\right)} + \underline{\cos x \left(\frac{1}{2}\right)} - \underline{\sin x \left(\frac{\sqrt{3}}{2}\right)}$$

$$0 + 1 \cos x \quad \text{∴}$$

14. $\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$

$$\frac{1}{2} [\overbrace{\cos x \cos y + \sin x \sin y} - \overbrace{(\cos x \cos y - \sin x \sin y)}]$$

$$\frac{1}{2} [\cancel{\cos x \cos y} + \cancel{\sin x \sin y} - \cancel{\cos x \cos y} + \cancel{\sin x \sin y}]$$

$$\frac{1}{2} (2 \sin x \sin y)$$

$$\sin x \sin y \quad \text{∴}$$

15. $(\cos A \cos B - \sin A \sin B)^2 + (\sin A \cos B + \cos A \sin B)^2 = 1$

Hint: Don't foil!

$$[\cos(A+B)]^2 + [\sin(A+B)]^2 = 1 \quad \text{✓} \qquad \sin^2 x + \cos^2 x = 1$$

Pythagorean Identity ∴