5.1 Trig Identities

Essential Question: How do I use trigonometric identities to transform expressions?

Reciprocal

$\cot x = \dfrac{1}{\tan x}$ or $\tan x \cot x = 1$ $\tan x = \dfrac{1}{\cot x}$

$\sec x = \dfrac{1}{\cos x}$ or $\cos x \sec x = 1$ $\cos x = \dfrac{1}{\sec x}$

$\csc x = \dfrac{1}{\sin x}$ or $\sin x \csc x = 1$ $\sin x = \dfrac{1}{\csc x}$

5.1 Trig Identities

Essential Question: How do I use trigonometric identities to transform expressions?

Quotient

- $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\sec x}{\csc x}$
- $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{\csc x}{\sec x}$

$\tan x = \dfrac{\sec x}{\csc x}$

$\dfrac{\sin x}{\cos x}$

$\dfrac{\sin x}{1} \cdot \dfrac{1}{\cos x}$

$\dfrac{1}{\csc x} \cdot \sec x$

$\dfrac{\sec x}{\csc x}$

5.1 Trig Identities

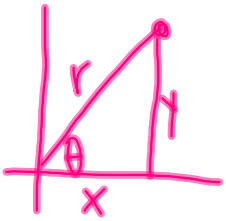
Essential Question: How do I use trigonometric identities to transform expressions?

Pythagorean

① $\cos^2 x + \sin^2 x = 1$

② $1 + \tan^2 x = \sec^2 x$

③ $\cot^2 x + 1 = \csc^2 x$



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$

$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

① $\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ ☺

② $1 + \tan^2\theta = \sec^2\theta$

③ $\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

5.1 Trig Identities

Essential Question: How do I use trigonometric identities to transform expressions?

1. sinxcotx to cosx

$\cancel{\sin x}\left(\frac{\cos x}{\cancel{\sin x}}\right)$

$\cos x$ ☺

2. sinxsecxcotx to 1

$\cancel{\sin x}\cdot\frac{1}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{\cancel{\sin x}}$ OR $\frac{1}{\cancel{\csc x}}\cdot\frac{\cancel{\sec x}}{1}\cdot\frac{\cancel{\csc x}}{\cancel{\sec x}}$

1 ☺ 1

3. $(\cos x - \sec x)^2$ to $\tan^2 x - \sin^2 x$

FOIL

$(\cos x - \sec x)(\cos x - \sec x)$

$\cos^2 x - \cos x\sec x - \cos x\sec x + \sec^2 x$

$\cos^2 x - 2(\cos x\sec x) + \sec^2 x$

$\cancel{1} - \sin^2 x - \cancel{2(1)} + \cancel{1} + \tan^2 x$

$\tan^2 x - \sin^2 x$ ☺

Pythag

$\cos^2 x + \sin^2 x = 1$

$-\sin^2 x \quad -\sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

4. $\cos^2 x - \sin^2 x$ to $1 - 2\sin^2 x$

$(1-\sin^2 x) - \sin^2 x$

$1 - 2\sin^2 x$ ☺

5. tanx + cotx to cscxsecx

$\left(\frac{\sin x}{\sin x}\right)\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\left(\frac{\cos x}{\cos x}\right)$

$\frac{\sin^2 x}{\sin x\cos x} + \frac{\cos^2 x}{\sin x\cos x} = \frac{\sin^2 x + \cos^2 x}{\sin x\cos x}$

$= \frac{1}{\sin x\cos x} = \frac{1}{\sin x}\cdot\frac{1}{\cos x} = \csc x\cdot\sec x$ ☺