5.10 Solving Trig Equations (with factoring) Name: Key

Solve each equation on the indicated domain, show all of your work!

1. $2\sin\theta\cos\theta = \sqrt{2}\cos\theta$

$\theta \in \{\text{real number degrees}\}$

$2\sin\theta\cos\theta - \sqrt{2}\cos\theta = 0$

$\cos\theta(2\sin\theta - \sqrt{2}) = 0$

$\cos\theta = 0$ $\quad$ $2\sin\theta - \sqrt{2} = 0$

$\boxed{90+360n,\ 270+360n}$ $\quad$ $\sin\theta = \frac{\sqrt{2}}{2}$

$\boxed{45+360n,\ 135+360n}$

2. $\tan x \sec x = \tan x$ $\quad$ $x \in \{\text{real numbers}\}$

$\tan x\sec x - \tan x = 0$

$\tan x(\sec x - 1) = 0$

$\tan x = 0$ $\quad$ $\sec x - 1 = 0$

$0 + \pi n$ $\quad$ $\sec x = 1$

$\cos^{-1}(1) = x$

$0 + 2\pi n$

$\boxed{\pi n}$

3. $2\sin^2 x + \sin x = 0$ $\quad$ $x \in (-\pi, \pi)$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0$ $\quad$ $2\sin x + 1 = 0$

$0 + \pi n$ $\quad$ $\sin x = -\frac{1}{2}$

$-\frac{\pi}{6} + 2\pi n$

$-\frac{5\pi}{6} + 2\pi n$

$\boxed{0, -\frac{\pi}{6}, -\frac{5\pi}{6}}$

4. $2\cos^2 x - 5\cos x + 2 = 0$ $\quad$ $x \in [0, 2\pi)$

$(2\cos^2 x - 4\cos x)(-1\cos x + 2) = 0$

$2\cos x(\cos x - 2) - 1(\cos x - 2) = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x - 2 = 0$

$\cos^{-1}(\frac{1}{2}) = x$ $\quad$ $\cos x = 2$

$\frac{\pi}{3} + 2\pi n,\ \frac{5\pi}{3} + 2\pi n$

$\boxed{\frac{\pi}{3}, \frac{5\pi}{3}}$

5. $2\sec^2 x - 3\sec x - 2 = 0$ $\quad$ $x \in [0, 2\pi)$

$(2\sec^2 x - 4\sec x)(+\sec x - 2) = 0$

$2\sec x(\sec x - 2) + 1(\sec x - 2) = 0$

$(2\sec x + 1)(\sec x - 2) = 0$

$2\sec x + 1 = 0$ $\quad$ $\sec x - 2 = 0$

$\sec x = -\frac{1}{2}$ $\quad$ $\sec x = 2$

NO SOLUTION

$\boxed{\frac{\pi}{3}, \frac{5\pi}{3}}$

6. $\sin^2 x + 5\sin x + 6 = 0$ $\quad$ $x \in [0, 2\pi)$

$(\sin^2 x + 2\sin x)(+3\sin x + 6) = 0$

$\sin x(\sin x + 2) + 3(\sin x + 2) = 0$

$(\sin x + 3)(\sin x + 2) = 0$

$\sin x + 3 = 0$ $\quad$ $\sin x + 2 = 0$

$\sin x = -3$ $\quad$ $\sin x = -2$

$\boxed{\text{NO SOLUTION}}$

7. $\tan^2 x - \sec x - 1 = 0$ $\quad$ $x \in [-\pi, \pi)$

$(\sec^2 x - 1) - \sec x - 1 = 0$

$\sec^2 x - \sec x - 2 = 0$

$(\sec^2 x - 2\sec x)(+\sec x - 2) = 0$

$\sec x(\sec x - 2) + 1(\sec x - 2) = 0$

$(\sec x + 1)(\sec x - 2) = 0$

$\sec x + 1 = 0$ $\quad$ $\sec x - 2 = 0$

$\sec x = -1$ $\quad$ $\sec x = 2$

$-\pi$ $\quad$ $\frac{\pi}{3} + 2\pi n$

$\frac{5\pi}{3} + 2\pi n$

$\boxed{-\pi, \frac{\pi}{3}, \frac{-\pi}{3}}$

8. $\tan^2 x + \tan x = 0$ $\quad$ $x \in (-\pi, \pi)$

$\tan x(\tan x + 1) = 0$

$\tan x = 0$ $\quad$ $\tan x + 1 = 0$

$0 + \pi n$ $\quad$ $\tan x = -1$

$-\frac{\pi}{4} + \pi n$

$\boxed{0, -\frac{\pi}{4}, \frac{3\pi}{4}}$

9. $4\csc^2 x + 4\csc x + 1 = 0$ $\quad$ $x \in [0, 2\pi)$

$(4\csc^2 x + 2\csc x)(+2\csc x + 1) = 0$

$2\csc x(2\csc x + 1) + 1(2\csc x + 1) = 0$

$\sqrt{(2\csc x + 1)^2} = \sqrt{0}$

$2\csc x + 1 = 0$

$\csc x = -\frac{1}{2}$

$\boxed{\text{NO SOLUTION}}$

10. $3 - 3\sin x - 2\cos^2 x = 0$ $\quad$ $x \in [-\pi, \pi]$

$3 - 3\sin x - 2(1 - \sin^2 x) = 0$

$3 - 3\sin x - 2 + 2\sin^2 x = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin^2 x - 2\sin x)(-1\sin x + 1) = 0$

$2\sin x(\sin x - 1) - 1(\sin x - 1) = 0$

$2\sin x - 1 = 0$ $\quad$ $\sin x - 1 = 0$

$\sin^{-1}(\frac{1}{2})$ $\quad$ $\sin^{-1}(1)$

$\frac{\pi}{6} + 2\pi n$ $\quad$ $\frac{\pi}{2} + 2\pi n$

$\frac{5\pi}{6} + 2\pi n$

$\boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}}$

11. $\sin 2x + \sqrt{3}\sin x = 0$ $\quad$ $x \in [0, 2\pi)$

$2\sin x\cos x + \sqrt{3}\sin x = 0$

$\sin x(2\cos x + \sqrt{3}) = 0$

$\sin x = 0$ $\quad$ $2\cos x + \sqrt{3} = 0$

$0 + \pi n$ $\quad$ $\cos x = -\frac{\sqrt{3}}{2}$

$\frac{5\pi}{6} + 2\pi n,\ \frac{7\pi}{6} + 2\pi n$

$\boxed{0, \frac{5\pi}{6}, \frac{7\pi}{6}, \pi}$

12. $4\sin^2 x + 7\sin x = 2$ $\quad$ $x \in [0, 2\pi)$

$4\sin^2 x + 7\sin x - 2 = 0$

$(4\sin^2 x + 8\sin x)(-\sin x - 2) = 0$

$4\sin x(\sin x + 2) - 1(\sin x + 2) = 0$

$(4\sin x - 1)(\sin x + 2) = 0$

$4\sin x - 1 = 0$ $\quad$ $\sin x = -2$

$\sin x = \frac{1}{4}$ $\quad$ NO SOLUTION

? have to use calculator