

Solve each equation on the indicated domain, show all of your work on a separate paper.

1. $4 \cos^2 \theta + 4 \cos \theta = -1$ $\theta \in [0^\circ, 360^\circ)$ $120^\circ, 240^\circ$
2. $\sin x \cos x = -\frac{1}{2}$ $x \in [0, 2\pi)$ $3\pi/4, 7\pi/4$
3. $2 \sin^2 x - 5 \sin x + 2 = 0$ $x \in [0, 2\pi)$ $\pi/6, 5\pi/6$
4. $\cos^2 x - \sin^2 x = 0$ $x \in [0, 2\pi)$ $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$
5. $4 \sin^2 \theta - 3 = 0$ $\theta \in [0^\circ, 360^\circ)$ $60^\circ, 120^\circ, 240^\circ, 300^\circ$
6. $\cos 2x = 2 - 2 \sin^2 x$ $x \in [0, 2\pi)$ NO SOLUTION
7. $\cos 4x \cos x + \sin 4x \sin x = -1$ $x \in [0, 2\pi)$ $\pi/3, \pi, 5\pi/3$
8. $\frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin x} = -\sqrt{3}$ $x \in \left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$ $-7\pi/6, -\pi/6, 5\pi/6$
9. $\tan(90^\circ - \theta) = -\frac{\sqrt{3}}{3}$ $\theta \in (-180^\circ, 180^\circ)$ $-60^\circ, 120^\circ$
10. $\sin 2\theta \cos 58^\circ + \cos 2\theta \sin 58^\circ = \frac{\sqrt{3}}{2}$ $\theta \in [0^\circ, 360^\circ)$ $1^\circ, 31^\circ, 181^\circ, 201^\circ$
11. $\cos 3\theta \cos 12^\circ - \sin 3\theta \sin 12^\circ = \frac{1}{2}$ $\theta \in (-180^\circ, 180^\circ)$ $-144^\circ, -104^\circ, -24^\circ, 16^\circ, 96^\circ, 136^\circ$
12. $\sin 2x = \cos x$ $x \in [0, 2\pi)$ $\pi/6, \pi/2, 5\pi/6, 3\pi/2$
13. $\tan 2(\theta + 41^\circ) = 1$ $\theta \in [0^\circ, 360^\circ)$ $71.5^\circ, 161.5^\circ, 251.5^\circ, 341.5^\circ$
14. $\sin \theta \cos 37^\circ = \cos \theta \sin 37^\circ$ $\theta \in [0^\circ, 360^\circ)$ $37^\circ, 217^\circ$
15. $\cos 2x - \sin x = 1$ $x \in [0, 2\pi)$ $0, \pi, 7\pi/6, 11\pi/6$
16. $\sin 2x + \cos x = 0$ $x \in [0, 2\pi)$ $\pi/2, 7\pi/6, 3\pi/2, 11\pi/6$
17. $4 \sin x \cos x = -\sqrt{3}$ $x \in (-\pi, \pi)$ $-\pi/3, -\pi/6, 2\pi/3, 5\pi/6$
18. $\cos 2x + \sin^2 x = 0$ $x \in [0, 2\pi)$ $\pi/2, 3\pi/2$
19. $2 \cos^2 x - 2 \cos 2x = 1$ $x \in [0, 2\pi)$ $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$
20. $\sin 2x - \cos x = 0$ $x \in [0, 2\pi)$ see #12
21. $\cos 2x + \cos x = 0$ $x \in [0, 2\pi)$ $\pi/3, \pi, 5\pi/3$
22. $(\sin x - \cos x)^2 = 1$ $x \in [0, 2\pi)$ $0, \pi/2, \pi, 3\pi/2$
23. $\sin x \cos x + \frac{1}{2} = 0$ $x \in [0, 2\pi)$ $3\pi/4, 7\pi/4$

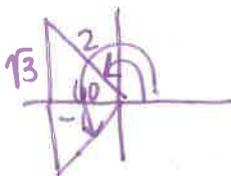
5.11 Mixed solving Trig Equations

① $4\cos^2\theta + 4\cos\theta + 1 = 0 \quad \theta \in [0, 360)$

$\frac{2}{4} \times \frac{4}{4} (4\cos^2\theta + 2\cos\theta)(2\cos\theta + 1) = 0$
 $2\cos\theta(2\cos\theta + 1) + 1(2\cos\theta + 1) = 0$
 $(2\cos\theta + 1)(2\cos\theta + 1) = 0$

$120^\circ, 240^\circ$

$2\cos\theta + 1 = 0$
 $\cos\theta = -\frac{1}{2}$
 $\cos^{-1}(-\frac{1}{2}) = \theta$



$120^\circ + 360n = \theta$
 $240^\circ + 360n = \theta$

② $2\sin x \cos x = -\frac{1}{2} \cdot 2 \quad x \in [0, 2\pi)$

$2\sin x \cos x = -1$

$\sin 2x = -1$

$\sin^{-1}(-1) = 2x$

$\frac{1}{2}(\frac{3\pi}{2} + 2\pi n) = \frac{2x}{2}$

$\frac{3\pi}{4} + \pi n = x$

$\frac{3\pi}{4}, \frac{7\pi}{4}$

③ $2\sin^2 x - 5\sin x + 2 = 0 \quad x \in [0, 2\pi)$

$\frac{4}{-4} \times \frac{-5}{-5} (2\sin^2 x - 4\sin x)(-\sin x + 2) = 0$
 $2\sin x(\sin x - 2) - 1(\sin x - 2) = 0$
 $(2\sin x - 1)(\sin x - 2) = 0$

$\frac{\pi}{6}, \frac{5\pi}{6}$

$2\sin x - 1 = 0$

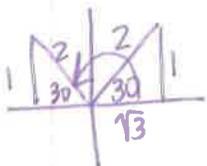
$\sin x = \frac{1}{2}$

$\sin^{-1}(\frac{1}{2}) = x$

$\sin x - 2 = 0$

$\sin x \neq 2$

DNE



$\frac{\pi}{6} + 2\pi n = x$

$\frac{5\pi}{6} + 2\pi n = x$

$$\textcircled{4} \quad \cos^2 x - \sin^2 x = 0 \quad x \in [0, 2\pi)$$

$$\cos(2x) = 0$$

$$\cos^{-1}(0) = 2x$$

$$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$\begin{array}{c} \sqrt{\text{CT}} \\ 0 \ 0 \ 1 \ 0 \\ \pi/2 \ 1 \ 0 \ 0 \\ \pi \ 0 \ -1 \ 0 \\ 3\pi/2 \ -1 \ 0 \ 0 \end{array}$	$\frac{1}{2} \left(\frac{\pi}{2} + 2\pi n \right) = \frac{2x}{2}$ $\frac{\pi}{4} + \pi n = x$	$\frac{1}{2} \left(\frac{3\pi}{2} + 2\pi n \right) = \frac{2x}{2}$ $\frac{3\pi}{4} + \pi n = x$
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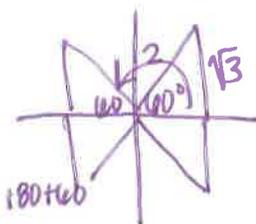
$$\textcircled{5} \quad 4\sin^2 \theta - 3 = 0 \quad \theta \in [0, 360)$$

$$60^\circ, 120^\circ, 240^\circ, 300^\circ$$

$$\sqrt{\sin^2 \theta} = \sqrt{\frac{3}{4}}$$

$$\sin \theta = \pm \frac{\sqrt{3}}{2}$$

$$\sin^{-1}\left(\pm \frac{\sqrt{3}}{2}\right) = \theta$$



$$\oplus 60^\circ + 360n$$

$$120^\circ + 360n$$

$$\ominus 240^\circ + 360n$$

$$300^\circ + 360n$$

$$\textcircled{6} \quad \cos 2x = 2 - 2\sin^2 x \quad x \in [0, 2\pi)$$

$$\cos 2x = 2(1 - \sin^2 x)$$

$$\cos 2x = 2\cos^2 x$$

$$2\cos^2 x - 1 = 2\cos^2 x$$

$$-2\cos^2 x \quad -2\cos^2 x$$

$-1 \neq 0$ **NO SOLUTION**

$$\textcircled{7} \quad \cos 4x \cos x + \sin 4x \sin x = -1 \quad x \in [0, 2\pi)$$

$$\cos(4x - x) = -1$$

$$\cos(3x) = -1$$

$$\cos^{-1}(-1) = 3x$$

$$\frac{1}{3} (\pi + 2\pi n) = \frac{3x}{3}$$

$$\frac{\pi}{3} + \frac{2\pi}{3} n = x$$

$$\frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$$\frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi$$

$$\frac{3\pi}{3} + \frac{2\pi}{3} = \frac{5\pi}{3}$$

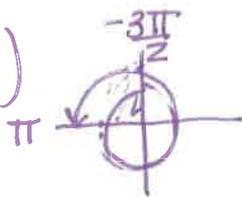
↙ cofunction

⑧ $\sin\left(\frac{\pi}{2} - x\right) = -\sqrt{3}$ $x \in \left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

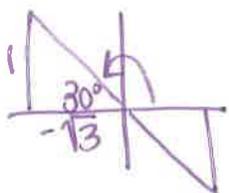
$$\frac{\sin x}{\cos x} = -\sqrt{3}$$

$$\cot x = -\sqrt{3}$$

$$\cot^{-1}(-\sqrt{3}) = x \quad \cot = \frac{\text{ADJ}}{\text{OPP}}$$



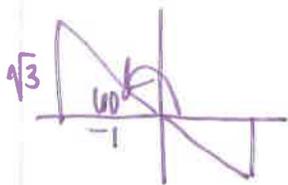
$$\boxed{\frac{5\pi}{6}, \frac{-7\pi}{6}, \frac{-\pi}{6}}$$



$$\frac{5\pi}{6} + \pi n = x$$

⑨ $\tan(90^\circ - \theta) = -\frac{\sqrt{3}}{3}$ $\theta \in (-180^\circ, 180^\circ)$ $\boxed{-60^\circ, 120^\circ}$

$$\cot \theta = -\frac{1}{\sqrt{3}}$$



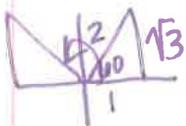
$$-60^\circ + 180n = \theta$$

⑩ $\sin 2\theta \cos 58^\circ + \cos 2\theta \sin 58^\circ = \frac{\sqrt{3}}{2}$ $\theta \in [0, 360)$

$$\sin(2\theta + 58) = \frac{\sqrt{3}}{2}$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 2\theta + 58$$

$$\boxed{1^\circ, 31^\circ, 181^\circ, 201^\circ}$$



$$\begin{matrix} 60^\circ + 360n = 2\theta + 58 \\ -58 \\ \hline \end{matrix}$$

$$\begin{matrix} 120^\circ + 360n = 2\theta + 58 \\ -58 \\ \hline \end{matrix}$$

$$\frac{1}{2}(2 + 360n) = \frac{2\theta}{2}$$

$$1 + 180n = \theta$$

$$\frac{1}{2}(62 + 360n) = \frac{2\theta}{2}$$

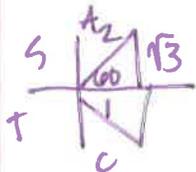
$$31 + 180n = \theta$$

$$(11) \cos 3\theta \cos 12^\circ - \sin 3\theta \sin 12^\circ = \frac{1}{2} \quad \theta \in (-180^\circ, 180^\circ)$$

$$\cos(3\theta + 12) = \frac{1}{2}$$

$$\cos^{-1}\left(\frac{1}{2}\right) = 3\theta + 12$$

$$\boxed{-144^\circ, -104^\circ, -24^\circ, 16^\circ, 96^\circ, 136^\circ}$$



$$\begin{array}{l|l} 60^\circ + 360n = 3\theta + 12 & -60^\circ + 360n = 3\theta + 12 \\ -12 & -12 \\ \hline \frac{1}{3}(48 + 360n) = \frac{3\theta}{3} & -72 + 360n = 3\theta \\ 16 + 120n = \theta & -24 + 120n = \theta \end{array}$$

$$(12) \sin 2x = \cos x \quad x \in [0, 2\pi)$$

$$2\sin x \cos x - \cos x = 0$$

$$\cos x (2\sin x - 1) = 0$$

$$\boxed{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}}$$

$$\begin{array}{l} \cos x = 0 \\ \cos^{-1}(0) = x \\ \frac{\pi}{2} + \pi n = x \end{array} \quad \begin{array}{l} 2\sin x - 1 = 0 \\ \sin^{-1}\left(\frac{1}{2}\right) = x \\ \frac{\pi}{6} + 2\pi n \\ \frac{5\pi}{6} + 2\pi n \end{array}$$

$$(13) \tan(2(\theta + 41^\circ)) = 1 \quad \theta \in [0, 360)$$

$$\tan^{-1}(1) = 2\theta + 82$$

$$45^\circ + 180n = 2\theta + 82$$

$$\begin{array}{r} -82 \\ -82 \end{array}$$

$$\frac{-37^\circ + 180n}{2} = \frac{2\theta}{2}$$

$$\boxed{71.5^\circ, 141.5^\circ, 251.5^\circ, 341.5^\circ}$$

$$-18.5^\circ + 90n = \theta$$

$$(14) \sin \theta \cos 37 - \cos \theta \sin 37 = 0 \quad \theta \in [0^\circ, 360^\circ)$$

$$\sin(\theta - 37^\circ) = 0$$

$$\sin^{-1}(0) = \theta - 37$$

$$\textcircled{1} \begin{array}{l} 0 + 360n = \theta - 37 \\ +37 \qquad \qquad +37 \end{array}$$

$$37 + 360n = \theta$$

$$\textcircled{2} \begin{array}{l} 180 + 360n = \theta - 37 \\ +37 \qquad \qquad +37 \end{array}$$

$$217 + 360n = \theta$$

$$\boxed{37^\circ, 217^\circ}$$

$$(15) \cos 2x - \sin x = 1 \quad x \in [0, 2\pi)$$

$$(1 - 2\sin^2 x) - \sin x = 1$$

$$\begin{array}{l} -1 \qquad \qquad \qquad -1 \\ -2\sin^2 x - \sin x = 0 \end{array}$$

$$-\sin x (2\sin x + 1) = 0$$

$$\frac{-\sin x}{-1} = \frac{0}{-1}$$

$$\sin^{-1}(0) = x$$

$$0 + 2\pi n = x$$

$$\pi + 2\pi n = x$$

$$2\sin x + 1 = 0$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = x$$



$$-\frac{\pi}{6} + 2\pi n = x$$

$$\frac{7\pi}{6} + 2\pi n = x$$

$$\boxed{0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

$$(16) \sin 2x + \cos x = 0 \quad x \in [0, 2\pi)$$

$$2\sin x \cos x + \cos x = 0$$

$$\cos x (2\sin x + 1) = 0$$

$$\cos x = 0$$

$$\cos^{-1}(0) = x$$

$$\frac{\pi}{2} + 2\pi n = x$$

$$\frac{3\pi}{2} + 2\pi n = x$$

$$2\sin x + 1 = 0$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = x$$

$$-\frac{\pi}{6} + 2\pi n$$

$$\frac{7\pi}{6} + 2\pi n$$

$$\boxed{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}}$$

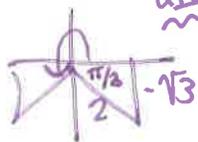
(17) $\frac{4 \sin x \cos x}{2} = -\frac{\sqrt{3}}{2} \quad x \in (-\pi, \pi)$

$2 \sin x \cos x = -\frac{\sqrt{3}}{2}$

$\sin 2x = -\frac{\sqrt{3}}{2}$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 2x$

$\frac{2\pi}{3} \approx \frac{1}{2} \left(-\frac{\pi}{3} + 2\pi n \right) = \frac{2x}{2}$
 $-\frac{\pi}{6} + \pi n = x$



$-\frac{\pi}{3}, -\frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}$

$\frac{2\pi}{3} \approx \frac{1}{2} \left(\frac{4\pi}{3} + 2\pi n \right) = \frac{2x}{2}$
 $\frac{2\pi}{3} + \pi n = x$

(18) $\cos 2x + \sin^2 x = 0 \quad x \in [0, 2\pi)$

$(1 - 2\sin^2 x) + \sin^2 x = 0$

$1 - \sin^2 x = 0$

$-\sin^2 x = -1$

$\sqrt{\sin^2 x} = \sqrt{1}$

$\sin x = \pm 1$

$\sin^{-1}(\pm 1) = x$

$\oplus \frac{\pi}{2} + 2\pi n = x$

$\ominus \frac{3\pi}{2} + 2\pi n = x$

$\frac{\pi}{2}, \frac{3\pi}{2}$

(19) $2 \cos^2 x - 2 \cos 2x = 1 \quad x \in [0, 2\pi)$

$2 \cos^2 x - 2(2 \cos^2 x - 1) = 1$

$2 \cos^2 x - 4 \cos^2 x + 2 = 1$

$\frac{-2 \cos^2 x}{-2} = \frac{-1}{-2}$

$\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}$

$\cos x = \pm \frac{1}{\sqrt{2}}$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\oplus \frac{\pi}{4} + 2\pi n = x$

$\ominus \frac{3\pi}{4} + 2\pi n = x$

$\frac{3\pi}{4} + 2\pi n = x$

$\frac{5\pi}{4} + 2\pi n = x$

20) $\sin 2x - \cos x = 0$
see #12

21) $\cos 2x + \cos x = 0 \quad x \in [0, 2\pi)$

$(2\cos^2 x - 1) + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$2 \times \frac{-2}{1} (2\cos^2 x + 2\cos x)(-1\cos x - 1) = 0$

$2\cos x (\cos x + 1) - 1(\cos x + 1) = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$\cos^{-1}(\frac{1}{2}) = x$

$\frac{\pi}{3} + 2\pi n = x$

$-\frac{\pi}{3} + 2\pi n = x$

$\cos x + 1 = 0$

$\cos^{-1}(-1) = x$

$\pi + 2\pi n = x$

22) $(\sin x - \cos x)^2 = 1 \quad x \in [0, 2\pi)$

$\sin^2 x - 2\sin x \cos x + \cos^2 x = 1$

$1 - \sin 2x = 1$

$-\sin 2x = 0$

$\sin^{-1}(0) = 2x$

$\frac{1}{2}(0 + \pi n) = 2x$
 $0 + \frac{\pi}{2}n$

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

23) $\sin x \cos x + \frac{1}{2} = 0 \quad x \in [0, 2\pi)$
 $\quad \quad \quad -\frac{1}{2} \quad -\frac{1}{2}$

$2 \cdot \sin x \cos x = -\frac{1}{2} \cdot 2$

$\sin 2x = -1$

$\sin^{-1}(-1) = 2x$

$\frac{1}{2}(\frac{3\pi}{2} + 2\pi n) = \frac{2}{2}x$

$\frac{3\pi}{4} + \pi n = x$

$\frac{3\pi}{4}, \frac{7\pi}{4}$