5.2 Trig Proofs

Essential Question:

How do I prove trig expressions are equivalent?

Before...

$\cos x \tan x$ to $\sin x$

Now...

$\cos x \tan x = \sin x$

Once you pick a side, you have to stick with that side!!!

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1. $\sec^2 x - \sin^2 x \sec^2 x = 1$

$\sec^2 x(1 - \sin^2 x)$

$\sec^2 x(\cos^2 x)$

$\frac{1}{\cos^2 x} \cdot \cos^2 x$

1 ☺

$\cos^2 x + \sin^2 x = 1$

$-\sin^2 x \quad -\sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

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Essential Question: How do I prove trig expressions are equivalent?

2. $1+2\tan^2 x = \sec^4 x - \tan^4 x$

$(\sec^2 x)^2 \quad (\tan^2 x)^2$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$1(\sec^2 x + \tan^2 x)$

$1 + \tan^2 x + \tan^2 x$

$1 + 2\tan^2 x$ ☺

diff. of squares

$x^2 - y^2$

$(x-y)(x+y)$

$\rightarrow 1 + \tan^2 x = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$

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3. $\csc x - \cot x = \dfrac{\sin x}{1+\cos x}$

$\dfrac{\sin x}{1+\cos x} \cdot \dfrac{1-\cos x}{1-\cos x}$

$\dfrac{\sin x(1-\cos x)}{(1+\cos x)(1-\cos x)}$

$\dfrac{\sin x(1-\cos x)}{1-\cos x+\cos x-\cos^2 x}$

$\dfrac{\sin x(1-\cos x)}{1-\cos^2 x}$

$\dfrac{\sin x(1-\cos x)}{\sin^2 x}$

$\dfrac{1-\cos x}{\sin x} = \dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}$

$\csc x - \cot x$ ☺

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$\dfrac{1}{1-\sqrt{3}} \cdot \dfrac{1+\sqrt{3}}{1+\sqrt{3}}$

$\dfrac{1}{3+i} \cdot \dfrac{3-i}{3-i}$

↑ sneaky form of 1

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Helpful techniques:

- Pick the side you wish to work on and write it down. It is usually easiest to start with the more complicated side.

- Look for algebraic things to do:
  - If there are two terms and you want only one:
    a.) add fractions
    b.) factor something out

  - Multiply by a clever form of 1:
    a.) to multiply a numerator or denominator by its conjugate
    b.) to get a desired expression in the numerator or denominator

  - Do any obvious algebra or arithmetic such as distributing, squaring, or multiplying polynomials

- Look for trigonometric things to do:
  - Search for familiar trigonometric expressions like
    $1-\cos^2 x$, $\cos x \bullet \sec x$, or $\frac{\sin x}{\cos x}$.
  - If there are squares of functions, think of Pythagorean properties.
  - Reduce the number of different functions, transforming them to the ones you want in your answer.

- Keep looking at the answer to make sure you are headed in the right direction.