

$$\textcircled{1} \quad \underline{\sec x(\sec x - \cos x)} = \tan^2 x$$

$$\sec x \sec x - \sec x \cos x$$

$$\sec^2 x - \frac{1}{\cos x} \cdot \cos x$$

$$\sec^2 x - 1 \quad (\text{Pythagorean Identity})$$

$$\tan^2 x \quad \text{QED}$$

$$\textcircled{3} \quad \underline{\sin x(\csc x - \sin x)} = \cos^2 x$$

$$\sin x \csc x - \sin^2 x$$

$$\sin x \cdot \frac{1}{\sin x} - \sin^2 x$$

$$1 - \sin^2 x$$

$$\cos^2 x \quad \text{QED}$$

$$\textcircled{5} \quad \underline{\csc^2 x - \cos^2 x \csc^2 x} = 1$$

$$\csc^2 x (1 - \cos^2 x)$$

$$\text{Pythag.} \\ \sin^2 x = 1 - \cos^2 x$$

$$\csc^2 x (\sin^2 x)$$

$$\frac{1}{\sin^2 x} \cdot \sin^2 x$$

$$1 \quad \text{QED}$$

$$\textcircled{7} \quad (\underline{\sec x + 1})(\underline{\sec x - 1}) = \tan^2 x \quad (a+b)(a-b) = a^2 - b^2$$

$$\sec^2 x - 1 \quad \text{Pythag.}$$

$$\tan^2 x \quad \text{QED}$$

$$\textcircled{9} \quad \underline{\sec^2 x + \tan^2 x} \sec^2 x = \sec^4 x$$

$$\sec^2 x (1 + \tan^2 x) \quad \text{Pythag.}$$

$$\sec^2 x \sec^2 x$$

$$\sec^4 x \quad \text{QED}$$

$$\textcircled{11} \quad \underline{\cos^4 x - \sin^4 x} = 1 - 2 \sin^2 x \quad \text{diff. of squares}$$

$$(cos^2 x + sin^2 x)(cos^2 x - sin^2 x)$$

$$\text{Pythag.}$$

$$1 (cos^2 x - sin^2 x)$$

sin in answer \rightarrow replace cos

$$(1 - \sin^2 x) - \sin^2 x$$

$$1 - 2 \sin^2 x \quad \text{QED}$$

HINTS

- pick more complicated side!

- want only one term?

 - ↳ Add (including fractions)

 - ↳ factor

- multiply by a sneaky 1

 - ↳ by conjugate

 - ↳ get what you want in bottom

- obvious algebra

 - ↳ distribute

 - ↳ factor

 - ↳ multiply

- use formula chart

 - ↳ look for trig identity

 - ↳ rearrange identities

- multiple correct solutions!

$$\textcircled{13} \quad \frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x} = \tan x$$

$$\frac{1 - \cos^2 x}{\sin x \cos x}$$

$$\frac{\sin^2 x}{\sin x \cos x}$$

$\frac{\sin x}{\cos x}$ Quotient Identity

$\tan x \text{ } \textcircled{13}$

$$\textcircled{15} \quad \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$$

$$\frac{\sin x}{1} + \frac{\cos x}{\cos x}$$

$$\sin x \cdot \frac{\sin x}{1} + \cos x \cdot \frac{\cos x}{1}$$

$$\sin^2 x + \cos^2 x \text{ Pythag.}$$

$1 \text{ } \textcircled{15}$

$$\textcircled{17} \quad \frac{1}{1 + \cos x} = \csc^2 x - \csc x \cot x$$

multiply by conjugate

OR
(right side)

$\csc x (\csc x - \cot x)$ Factor

$\frac{1}{\sin x} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$ Quot Recip.

$$\frac{1}{\sin x} \left(\frac{1 - \cos x}{\sin x} \right)$$

$$\frac{1 - \cos x}{\sin^2 x}$$

$$\frac{1 - \cos x}{1 - \cos^2 x}$$

← diff. of squares

$$\frac{1 - \cos x}{(1 - \cos x)(1 + \cos x)}$$

$$\frac{1}{1 + \cos x} \text{ } \textcircled{17}$$

$$\frac{1 - \cos x}{1 - \cos x (1 + \cos x)}$$

$$\frac{1 - \cos x}{1 - \cos^2 x}$$

$$\frac{1 - \cos x}{\sin^2 x} \text{ Pythag.}$$

$$\frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x}$$

$$\csc^2 x - \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} \text{ Reciprocal}$$

$$\csc^2 x - \cot x \csc x \text{ Quotient/Recip.}$$

$\textcircled{17}$

$$\textcircled{19} \quad \frac{\cos x}{\sec x - 1} - \frac{\cos x}{\tan^2 x} = \cot^2 x$$

$$\frac{\cos x}{\sec x - 1} - \frac{\cos x}{\sec^2 x - 1} \quad \sec^2 x - 1 = (\sec x - 1)(\sec x + 1)$$

$$\frac{\cos x(\sec x + 1) - \cos x}{\sec^2 x - 1}$$

$$\frac{\cos x[(\sec x + 1) - 1]}{\sec^2 x - 1}$$

$$\frac{\cos x \sec x}{\sec^2 x - 1}$$

$$\frac{\cos x \cdot \frac{1}{\cos x}}{\sec^2 x - 1}$$

$$\frac{1}{\sec^2 x - 1}$$

$$\frac{1}{\tan^2 x}$$

$$\cot^2 x \quad \text{☺}$$

$$\textcircled{21} \quad \frac{\sec x}{\sec x - \tan x} = \sec^2 x + \sec x \tan x$$

$$\frac{\sec x + \tan x}{\sec x - \tan x} \cdot \frac{\sec x}{\sec x - \tan x}$$

$$\frac{\sec x(\sec x + \tan x)}{\sec^2 x - \tan^2 x}$$

$$\frac{\sec^2 x + \sec x \tan x}{1}$$



$$\textcircled{23} \quad \sin^3 x \cos^2 x = \underline{\sin^3 x - \sin^5 x}$$

$$\frac{\sin^3 x(1 - \underline{\sin^2 x})}{\sin^3 x \cdot \cos^2 x} \quad \text{Pythag}$$

$$\textcircled{25} \quad \underline{\sec^2 x + \csc^2 x} = \sec^2 x \csc^2 x$$

$$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$$

$$\frac{1}{\cos^2 x \sin^2 x}$$

$$\frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x}$$

$$\sec^2 x \csc^2 x \quad \text{☺}$$

$$\textcircled{27} \quad \underline{\frac{1-3\cos x - 4\cos^3 x}{\sin^2 x}} = \frac{1-4\cos x}{1-\cos x}$$

$$\frac{1-3x-4x^2}{(1-4x)(1+x)}$$

$$\frac{(1-4\cos x)(1+\cos x)}{1-\cos^2 x}$$

$$\frac{(1-4\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x)}$$

$$\frac{1-4\cos x}{1-\cos x} \quad \text{☺}$$