5.3 Odd/Even, Cofunction, and Sum & Difference Properties

Essential Question:

How do I use trig properties to simplify expressions & prove identities?

Odd-Even

$\sin(-x) = -\sin x$ (odd) $\csc(-x) = -\csc x$ (odd)

$\cos(-x) = \cos x$ (even) $\sec(-x) = \sec x$ (even)

$\tan(-x) = -\tan x$ (odd) $\cot(-x) = -\cot x$ (odd)

Co-function

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$ $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$ $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\sec\left(\frac{\pi}{2} - x\right) = \csc x$ $\csc\left(\frac{\pi}{2} - x\right) = \sec x$

Composite Argument (Sum and Difference)

$\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

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Odd/Even Property

EVEN

$f(-x) = f(x)$

cos sec

$f(x) = x^2$

$f(-2) = 4$
$f(2) = 4$ > same!

$\cos(-30^\circ) =$ $\cos 30^\circ$

ODD

$f(-x) = -f(x)$

sin tan
csc cot

$\sin(-\frac{\pi}{2}) =$ $-\sin(\frac{\pi}{2})$

Odd-Even

$\sin(-x) = -\sin x$	(odd)	$\csc(-x) = -\csc x$	(odd)
$\cos(-x) = \cos x$	(even)	$\sec(-x) = \sec x$	(even)
$\tan(-x) = -\tan x$	(odd)	$\cot(-x) = -\cot x$	(odd)

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Cofunction Property

complement 90°

sin ⟷ cos

csc ⟷ sec

tan ⟷ cot

Co-function

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x \qquad \cos\left(\frac{\pi}{2}-x\right)=\sin x$$

$$\tan\left(\frac{\pi}{2}-x\right)=\cot x \qquad \cot\left(\frac{\pi}{2}-x\right)=\tan x$$

$$\sec\left(\frac{\pi}{2}-x\right)=\csc x \qquad \csc\left(\frac{\pi}{2}-x\right)=\sec x$$

$\frac{\pi}{2}=90°$

ex. $\sin 30° = \cos(90-30)$

$= \cos 60$

$\frac{1}{2}$ $\frac{1}{2}$

ex. $\sec 42° = \csc(90-42)$

$= \boxed{\csc 48°}$

ex. $\cot\frac{3\pi}{8} = \tan\left(\frac{\pi}{2}-\frac{3\pi}{8}\right)$

$= \tan\left(\frac{4\pi}{8}-\frac{3\pi}{8}\right)$

$= \tan\frac{\pi}{8}$

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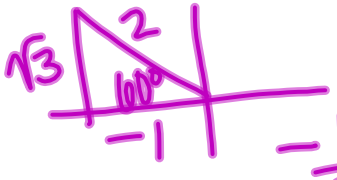
Sum & Difference Properties

$$\cos(A+B) \neq \cos A + \cos B$$

$\cos 120°$ $\qquad$ $\cos 30 + \cos 90$



$-\frac{1}{2} \neq \frac{\sqrt{3}}{2} + 0$

$A = 30°$
$B = 90°$

$(x+y)^2 \neq x^2 + y^2$

Composite Argument (Sum and Difference)

$$\cos(A+B) = \cos A\cos B - \sin A\sin B \qquad \cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B \qquad \sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

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Sum & Difference Properties

Composite Argument (Sum and Difference)

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$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

ex. Simplify $\cos\underset{A}{34^\circ}\cos\underset{B}{20^\circ} + \sin 34^\circ\sin 20^\circ$

$\cos(A-B) = \cos(34-20) = \cos(14)^\circ$

Prove:

ex. $\cos(\underset{A}{x} - \underset{B}{\pi}) = -\cos x$

expand

$\cos x\,\boxed{\cos\pi} + \sin x\,\boxed{\sin\pi}$

$\cos x\,(-1) + \cancel{\sin x\,(0)}$

$-\cos x$ ☺

$(-1,0)$

ex. $\csc(\theta - 90^\circ) = -\sec\theta$

$A=\theta$
$B=90$

$\dfrac{1}{\sin(\theta-90)}$

$\dfrac{1}{\cancel{\sin\theta\cos 90} - \cos\theta\sin 90}$

0 1

$\dfrac{1}{-\cos\theta} = -\sec\theta$ ☺

$(0,1)$