

5.1 Reciprocal, Quotient, and Pythagorean Identities

1. $\cos x \tan x$ to $\sin x$

$$\frac{\cos x \cdot \frac{\sin x}{\cos x}}{\sin x}$$

$$\sin x \quad \text{ü}$$

3. $\sec x \cot x \sin x$ to 1

$$\frac{1}{\cos x} \cdot \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} \cdot \frac{\sin x}{1}$$

$$1 \quad \text{ü}$$

5. $\sin^2 x \sec x \csc x$ to $\tan x$

$$\frac{\sin^2 x}{\cos x} \cdot \frac{1}{\sin x} \cdot \frac{1}{\cos x}$$

$$\frac{\sin x}{\cos x}$$

$$\tan x \quad \text{ü}$$

7. $\tan x + \cot x$ to $\csc x \sec x$

$$\left(\frac{\sin x}{\cos x} \right) \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \left(\frac{\cos x}{\sin x} \right)$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x}$$

$$\frac{1}{\sin x} \cdot \frac{1}{\cos x}$$

$$\csc x \sec x \quad \text{ü}$$

9. $\csc x - \sin x$ to $\cot x \cos x$

$$\frac{1}{\sin x} - \sin x \left(\frac{\sin x}{\sin x} \right)$$

$$\frac{1 - \sin^2 x}{\sin x}$$

$$\frac{\cos^2 x}{\sin x}$$

$$\frac{\cos x}{\sin x}$$

$$\cos x \left(\frac{\cos x}{\sin x} \right)$$

$$\cos x \cot x \quad \text{QED}$$

11. $\tan x (\sin x + \cot x \cos x)$ to $\sec x$

$$\frac{\sin x}{\cos x} \left(\sin x + \frac{\cos x}{\sin x} \cdot \cos x \right)$$

$$\frac{\sin^2 x}{\cos x} + \frac{\sin x \cos^2 x}{\sin x \cos x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$\frac{1}{\cos x}$$

$$\sec x \quad \text{QED}$$

13. $(1 + \sin x)(1 - \sin x)$ to $\cos^2 x$

$$1 - \sin x + \sin x - \sin^2 x$$

$$1 - \sin^2 x$$

$$\cos^2 x \quad \text{QED}$$

15. $(\cos x - \sin x)^2$ to $1 - 2\cos x \sin x$

$$(\cos x - \sin x)(\cos x - \sin x)$$

$$\cos^2 x - \cos x \sin x - \cos x \sin x + \sin^2 x$$

$$(\cos^2 x + \sin^2 x) - 2 \cos x \sin x$$

$$1 - 2 \cos x \sin x \quad \text{QED}$$

$$17. (\tan x + \cot x)^2 \text{ to } \sec^2 x + \csc^2 x$$

$$(\tan x + \cot x)(\tan x + \cot x)$$

$$\tan^2 x + 2 \cot x \tan x + \cot^2 x$$

$$\sec^2 x - 1 + 2 \cancel{\cot x \tan x} + \csc^2 x - 1$$

$$\sec^2 x + \csc^2 x - 1 + 2(1) - 1$$

$$\sec^2 x + \csc^2 x \quad \text{ü}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

$$\frac{\cos^2 x + \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\cot^2 x = \csc^2 x - 1$$

$$19. \frac{\csc^2 x - 1}{\cos x} \text{ to } \cot x \csc x$$

$$\frac{\cot^2 x}{\cos x}$$

$$\frac{\cos^2 x}{\sin^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos x}$$

$$\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$21. \frac{\sec^2 x - 1}{\sin x} \text{ to } \tan x \sec x$$

$$\frac{\tan^2 x}{\sin x}$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\tan x \sec x \quad \text{ü}$$

$$23. \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} \quad \text{to } \cot x$$

$$\frac{1}{\cos x \sin x} - \frac{\sin x}{\cos x} \left(\frac{\sin x}{\sin x} \right)$$

$$\frac{1 - \sin^2 x}{\cos x \sin x}$$

$$\frac{\cos^2 x}{\cos x \sin x}$$

$$\frac{\cos x}{\sin x}$$

$\cot x \text{ } \textcircled{u}$

$$25. \frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} \quad \text{to } 2\csc^2 x$$

$$\frac{1 + \cos x + 1 - \cos x}{(1 - \cos x)(1 + \cos x)}$$

$$\frac{2}{1 - \cos^2 x}$$

$$\frac{2}{\sin^2 x}$$

$2 \csc^2 x \text{ } \textcircled{u}$