Your work is your answer.

5.4 Proving Trig Identities **Name:**_____

Prove each identity. You may start on either side, but once you start you must work only on one side! Use a separate sheet of paper.

1. $\sec x(\sec x - \cos x) = \tan^2 x$

$\sec x(\sec x - \cos x)$

$\sec^2 x - \sec x \cos x$

$\sec^2 x - 1$

$(1 + \tan^2 x) - 1$

$\tan^2 x$ ☺

2. $\tan x(\cot x + \tan x) = \sec^2 x$

$\tan x \cot x + \tan^2 x$

$1 + \tan^2 x$ Pythag.

$\sec^2 x$

☺

3. $\sin x(\csc x - \sin x) = \cos^2 x$

$\sin x \csc x - \sin^2 x$

$1 - \sin^2 x$

$\cos^2 x + \cancel{\sin^2 x} - \cancel{\sin^2 x}$

$\cos^2 x$ ☺

4. $\cos x(\sec x - \cos x) = \sin^2 x$

$\cos x \sec x - \cos^2 x$

$1 - \cos^2 x$

$\cancel{\cos^2 x} + \sin^2 x - \cancel{\cos^2 x}$

$\sin^2 x$ ☺

5. $\csc^2 x - \cos^2 x \csc^2 x = 1$

$\frac{1}{\sin^2 x} - \frac{\cos^2 x}{1} \cdot \frac{1}{\sin^2 x}$ Already has common denom.

$\frac{1 - \cos^2 x}{\sin^2 x}$

$\frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}} = 1$ ☺

6. $\cos^2 x + \tan^2 x \cos^2 x = 1$

$\cos^2 x + \frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x}$

$\cos^2 x + \sin^2 x$ Pythag

1 ☺

7. $(\sec x + 1)(\sec x - 1) = \tan^2 x$

$\sec^2 x - \cancel{\sec x} + \cancel{\sec x} - 1$

$\sec^2 x - 1$

$1 + \tan^2 x - 1$

$\tan^2 x$ ☺

8. $(1 + \sin x)(1 - \sin x) = \cos^2 x$

$1 - \sin^2 x$

$\cos^2 x + \cancel{\sin^2 x} - \cancel{\sin^2 x}$

$\cos^2 x$ ☺

9. $\sec^2 x + \tan^2 x \sec^2 x = \sec^4 x$

$\sec^2 x(1+\tan^2 x)$

$\sec^2 x(\sec^2 x)$

$\sec^4 x$ ☺

10. $\cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$

$\cot^2 x(\csc^2 x - 1)$

$\cot^2 x(\cot^2 x + \cancel{1} - \cancel{1})$

$\cot^2 x \cdot \cot^2 x$

$\cot^4 x$ ☺

11. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

difference of squares

$(\cos^2 x - \sin^2 x)(\underbrace{\cos^2 x + \sin^2 x}_{1})$

$\cos^2 x + \sin^2 x = 1$

$-\sin^2 x - \sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

$1 - \sin^2 x - \sin^2 x$

$1 - 2\sin^2 x$ ☺

12. $\sec^4 x - \tan^4 x = 1 + 2\tan^2 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$(1 + \cancel{\tan^2 x} - \cancel{\tan^2 x})(1 + \tan^2 x + \tan^2 x)$

$(1)(1 + 2\tan^2 x)$

$1 + 2\tan^2 x$ ☺

13. $\dfrac{1}{\sin x \cos x} - \dfrac{\cos x}{\sin x} \cdot \dfrac{\cos x}{\cos x} = \tan x$

CD: $\sin x \cos x$

$\dfrac{1}{\sin x \cos x} - \dfrac{\cos^2 x}{\sin x \cos x}$

$\dfrac{1 - \cos^2 x}{\sin x \cos x}$

$\dfrac{\sin^2 x}{\sin x \cos x} = \dfrac{\cancel{\sin x} \sin x}{\cancel{\sin x} \cos x}$

$\tan x$ ☺

14. $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x} = \cot x$

CD: $\sin x \cos x$

$\dfrac{\cos x}{\cos x} \cdot \dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x} \cdot \dfrac{\sin x}{\sin x}$

$\dfrac{\cos x \sec x - \sin^2 x}{\cos x \sin x}$

$\dfrac{1 - \sin^2 x}{\cos x \sin x}$

$\dfrac{\cos^2 x}{\cos x \sin x} = \dfrac{\cos x}{\sin x}$

$\cot x$ ☺

15. $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

$\dfrac{\sin x}{\frac{1}{\sin x}} + \dfrac{\cos x}{\frac{1}{\cos x}}$

$\sin^2 x + \cos^2 x$

1 ☺

16. $\dfrac{1}{\sec^2 x} + \dfrac{1}{\csc^2 x} = 1$

$\cos^2 x + \sin^2 x$

☺