

5.7 Double Angle Properties

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How do I use double angle properties to simplify expressions and solve equations?

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$$1. \sin 2x = \sin(x + x)$$

$$\begin{aligned} & \sin x \cos x + \cos x \sin x \\ &= 2 \sin x \cos x \quad (\checkmark) \end{aligned}$$

$$2. \cos 2x = \cos(x + x)$$

$$\begin{aligned} & \cos x \cos x - \sin x \sin x \\ & \textcircled{1} \cos^2 x - \sin^2 x \\ & \textcircled{2} (1 - \sin^2 x) - \sin^2 x \\ & \textcircled{3} \cos^2 x - (1 - \cos^2 x) \\ & \cos^2 x - 1 + \cos^2 x = 2 \cos^2 x - 1 \end{aligned}$$

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x \\ \sin^2 x &= 1 - \cos^2 x \end{aligned}$$

$$3. \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\frac{\sin 2x}{\cos 2x}$$

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Double Argument

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

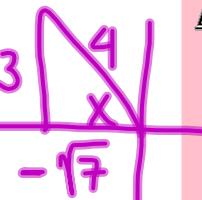
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ex. $\sin x = \frac{3}{4}$

$$\frac{\pi}{2} < x < \pi$$



find $\sin 2x$, $\cos 2x$, $\tan 2x$

Double Argument

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ &= 2 \left(\frac{3}{4}\right) \left(-\frac{\sqrt{7}}{4}\right) = -\frac{6\sqrt{7}}{16} = \boxed{-\frac{3\sqrt{7}}{8}}\end{aligned}$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= \left(-\frac{\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2 \\ &= \frac{7}{16} - \frac{9}{16} = \frac{-2}{16} = \boxed{-\frac{1}{8}}\end{aligned}$$

$$\begin{aligned}\tan 2x &= \frac{\sin 2x}{\cos 2x} = \frac{-\frac{3\sqrt{7}}{8}}{-\frac{1}{8}} = \frac{3\sqrt{7}}{8} \cdot \frac{8}{1} = \boxed{3\sqrt{7}}\end{aligned}$$

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ex. A. General Solution

B. Values $0 \leq x \leq 2\pi$

Double Argument

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\frac{4 \sin x \cos x}{2} = \frac{2}{2}$$

$$2 \sin x \cos x = 1$$

$$\sin 2x = 1$$

$$\sin^{-1}(1) = 2x$$

SCT
0 0 10
90 0 0
180 0 -10
270 -10

$$\frac{1}{2} \left(\frac{\pi}{2} + 2\pi n \right) = \frac{2x}{2}$$

$$\frac{\pi}{4} + \pi n = x$$

B. $\frac{\pi}{4}, \frac{5\pi}{4}$

$$\frac{\pi}{4} + \frac{4\pi}{4} = \frac{5\pi}{4} +$$

$$\frac{\pi}{4} - \frac{4\pi}{4} = \frac{3\pi}{4}, \frac{9\pi}{4}$$

general solution A.