5.8 Half Angle Properties

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1. Simplify $\sqrt{\frac{1}{2}(1-\cos 4A)}$

x

$\sin\left(\frac{1}{2}(\overset{2}{\cancel{4}}A)\right) = \boxed{\sin 2A}$

Half Argument

$$\sin\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1-\cos x)}$$

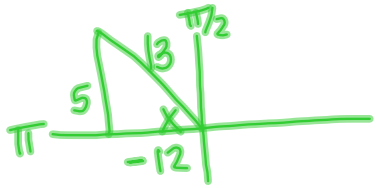
$$\cos\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1+\cos x)}$$

$$\tan\frac{1}{2}x = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

2. $\csc x = \frac{13}{5}$ $\quad \frac{\pi}{2} < x < \pi$ QII

Find $\sin\frac{1}{2}x$, $\cos\frac{1}{2}x$, $\tan\frac{1}{2}x$

Picture



$\cos x = \frac{-12}{13}$

$\sin\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1-\cos x)}$

$= \pm\sqrt{\frac{1}{2}\left(1-\left(-\frac{12}{13}\right)\right)}$

$= \pm\sqrt{\frac{1}{2}\left(\frac{13}{13}+\frac{12}{13}\right)}$

$= \pm\sqrt{\frac{1}{2}\left(\frac{25}{13}\right)}$

$= \pm\sqrt{\frac{25}{26}}$

$\frac{\pi}{2} < \frac{x}{2} < \frac{\pi}{2}$

$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

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$= \pm\frac{5}{\sqrt{26}}$

$\boxed{\sin\frac{1}{2}x = \frac{5}{\sqrt{26}}}$

$\cos\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1+\cos x)}$

$= \pm\sqrt{\frac{1}{2}\left(1+\frac{-12}{13}\right)}$

$= \pm\sqrt{\frac{1}{2}\left(\frac{13}{13}-\frac{12}{13}\right)}$

$= \pm\sqrt{\frac{1}{2}\cdot\frac{1}{13}} = \pm\sqrt{\frac{1}{26}}$

$\cos\frac{1}{2}x = \boxed{\frac{1}{\sqrt{26}}}$

$\tan\frac{1}{2}x = \frac{\sin\frac{1}{2}x}{\cos\frac{1}{2}x} = \frac{5}{\sqrt{26}} \div \frac{1}{\sqrt{26}} = \frac{5}{\cancel{\sqrt{26}}}\cdot\frac{\cancel{\sqrt{26}}}{1} = \boxed{5}$

3. If $\cos x = -\frac{1}{9}$ with $180° < x < 270°$, find $\cos\left(\frac{x}{2}\right)$

$= \boxed{-\frac{2}{3}}$

$\cos\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1+\cos x)}$

$= \pm\sqrt{\frac{1}{2}\left(\frac{9}{9}+\left(\frac{-1}{9}\right)\right)}$

$= \pm\sqrt{\frac{1}{2}\left(\frac{8}{9}\right)}$

$= \pm\sqrt{\frac{8}{18}} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$\frac{180}{2} < \frac{x}{2} < \frac{270}{2}$

$90 < \frac{x}{2} < 135$ QII