Name:_______________

Factor and Remainder Theorem

Find the zeros of the given polynomials algebraically by factoring and setting each factor equal to zero. Check your answers with your calculator.

1. $f(x) = x^2 + 7x$

$x = 0,\ x = -7$

2. $f(a) = (a+3)^2 - 64$

$a = 5,\ a = -11$

3. $h(x) = 6x^2 + x - 1$

$x = -1/2 \quad x = 1/3$

4. $k(y) = 9y^4 + 3y^3 - 6y^2$

$y = 0,\ y = 2/3,\ y = -1$

5. $h(x) = x^3 - 5x^2 - x + 5$

$x = 5,\ x = 1,\ x = -1$

6. $f(x) = 3x^3 + 4x^2 - 27x - 36$

$x = 3,\ x = -3,\ x = -4/3$

$x^2(3x+4) - 9(3x+4)$

$(x^2 - 9)(3x+4)$

$(x+3)(x-3)(3x+4)$

Find the remainder when $f(x)$ is divided by $g(x)$, without using division.

7. $f(x) = x^{10} + x^8$; $\qquad g(x) = x - 1$

2

8. $f(x) = x^6 - 10$; $\qquad g(x) = x - 2$

54

Use the Factor Theorem to determine whether $h(x)$ is a factor of $f(x)$.

9. $h(x) = x - 1 \qquad f(x) = x^5 + 1$

no

10. $h(x) = x + 1 \qquad f(x) = 14x^{99} + 65x^{56} - 51$

yes

11. Find a polynomial of degree 4 such that $f(3) = 288$ and the zeros of $f(x)$ are 0, -1, 2, and -3.

$4x(x+1)(x-2)(x+3)$

12. Find a number *k* such that $x - 1$ is a factor of $k^2x^4 - 2kx^2 + 1$.

$k = 1$

$k^2(1) - 2k(1) + 1 = 0$

$k^2 - 2k + 1 = 0$

$(k-1)^2 = 0$

$\boxed{k = 1}$

13. When $x^3 + cx + 4$ is divided by $x + 2$, the remainder is 4. Find *c*.

-4

Use synthetic division and the remainder theorem to evaluate P(c)

14. $P(\mathrm{x}) = 4\mathrm{x}^2 + 12x + 5 \quad c = -1$

-3

15. $P(\mathrm{x}) = \mathrm{x}^3 + 2x^2 - 7 \quad c = 2$

9

16. Given that 3 is a zero of the polynomial $P(\mathrm{x}) = x^3 + 2x^2 - 11x - 12$, factor completely

$$\begin{array}{r|rrrr} 3 & 1 & 2 & -11 & -12 \\ & \downarrow & 3 & 15 & 12 \\ \hline & 1 & 5 & 4 & \boxed{0} \end{array}$$

$(x-3)(x+4)(x+1)$

$x^2 + 5x + 4$

$(x+4)(x+1)$

17. Given that 2 is a zero of the polynomial $P(\mathrm{x}) = x^3 - 4x^2 - 11x + 30$, factor completely

$(x-5)(x+3)(x-2)$

18.

Impossible Division? Suppose you were asked to solve the following two problems on a test:

A. Find the remainder when $6x^{1000} - 17x^{562} + 12x + 26$ is divided by $x + 1$.

B. Is $x - 1$ a factor of $x^{567} - 3x^{400} + x^9 + 2$?

Obviously, it's impossible to solve these problems by dividing, because the polynomials are of such large degree. Use one or more of the theorems in this section to solve these problems *without* actually dividing.

A. 3

B. No