

5.5

Sum and Difference Properties (3)

Name: KEY

For each equation, find:

- a) the general solution (2 equations with n)
 b) the particular values for $0 \leq x < 2\pi$ or $0 \leq \theta < 360^\circ$

1. $\cos x \cdot \cos \frac{\pi}{5} - \sin x \cdot \sin \frac{\pi}{5} = \frac{\sqrt{3}}{2}$

$$\cos(x + \frac{\pi}{5}) = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}(\frac{\sqrt{3}}{2}) = x + \frac{\pi}{5}$$

B. $\frac{-\pi}{30} + \frac{60\pi}{30} = \frac{59\pi}{30}$
 $-\frac{11\pi}{30} + \frac{60\pi}{30} = \frac{49\pi}{30}$

QI $\frac{2\pi}{15}$ | $\frac{\pi}{6} + 2\pi n = x + \frac{\pi}{5}$
 $-\frac{\pi}{5}$

$$\frac{5\pi}{30} - \frac{6\pi}{30} = -\frac{\pi}{30}$$

A. $-\frac{\pi}{30} + 2\pi n = x$

QIV $\frac{\pi}{2} - \frac{\sqrt{3}}{2}$

$$-\frac{\pi}{6} + 2\pi n = x + \frac{\pi}{5}$$

$$-\frac{5\pi}{30} - \frac{6\pi}{30}$$

$$-\frac{11\pi}{30} + 2\pi n = x$$

2. $\sin \theta \cdot \cos 35^\circ + \cos \theta \cdot \sin 35^\circ = \frac{1}{2}$

$$\sin(\theta + 35^\circ) = \frac{1}{2}$$

$$\sin^{-1}(\frac{1}{2}) = \theta + 35^\circ$$

B. $115^\circ, 355^\circ$
 $-5 + 360^\circ$

QI $\frac{2\pi}{9}$ | $30^\circ + 360^\circ n = \theta + 35^\circ$
 -35°

QII $\frac{11\pi}{15}$ | $180^\circ - 30^\circ$

A. $-5^\circ + 360^\circ n = \theta$
 $115^\circ + 360^\circ n = \theta$

3. $\sin 2\theta \cdot \cos \theta - \cos 2\theta \cdot \sin \theta = -\frac{\sqrt{2}}{2}$

$$\sin(2\theta - \theta) = -\frac{\sqrt{2}}{2}$$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

B. $225^\circ, 315^\circ$
 $-45 + 360^\circ$

QIV $\frac{\pi}{4}$ | $x^2 + (-\sqrt{2})^2 = 2^2$
 $x^2 = 2$

QIII $\frac{5\pi}{4}$ | $180^\circ + 45^\circ$

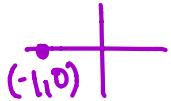
A. $-45 + 360n = \theta$
 $225 + 360n = \theta$

4. $\cos 2x \cdot \cos x + \sin 2x \cdot \sin x = -1$

$$\cos(2x-x) = -1$$

$$\cos x = -1$$

$$\cos^{-1}(-1) = x$$



A. $x = \pi + 2\pi n$

B. π

5. $\frac{\tan 2x - \tan x}{1 + \tan 2x \cdot \tan x} = \sqrt{3}$

$$\tan(2x-x) = \sqrt{3}$$

$$\tan x = \sqrt{3}$$

$$\tan^{-1}\sqrt{3} = x$$

QI $\frac{2\sqrt{3}}{1} \Rightarrow \theta = \frac{\pi}{3}$ A. $\frac{\pi}{3} + \pi n$

B. $\frac{\pi}{3}, \frac{4\pi}{3}$



6. $\frac{\tan \theta + \tan 27^\circ}{1 - \tan \theta \cdot \tan 27^\circ} = 1$

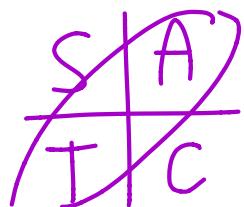
$$\tan(\theta + 27^\circ) = 1$$

$$\tan^{-1}(1) = \theta + 27^\circ$$

QI $\frac{1}{1} \Rightarrow \theta = 45^\circ$ $45 + 180n = \theta + 27^\circ$

A. $18 + 180n = \theta$

B. $18^\circ, 198^\circ$
 $18 + 180$



Prove:

$$7. \sin\left(x + \frac{\pi}{3}\right) - \cos\left(x + \frac{\pi}{6}\right) = \sin x$$

$$\left(\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3}\right) - \left(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}\right)$$

$$\sin x \left(\frac{1}{2}\right) + \cos x \left(\frac{\sqrt{3}}{2}\right) - \left(\cos x \left(\frac{\sqrt{3}}{2}\right) - \sin x \left(\frac{1}{2}\right)\right)$$

$$\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x - \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x$$

$$\frac{1}{2} \sin x + \frac{1}{2} \sin x$$

$$1 \sin x \quad \text{Q.E.D.}$$

$$8. \tan\left(x + \frac{\pi}{4}\right) + 1 = \sqrt{2} \cos x \cdot \sec\left(x + \frac{\pi}{4}\right)$$

$$\frac{\tan x + \tan \frac{\pi}{4}}{1 - \tan x \tan \frac{\pi}{4}} + 1$$

$$\frac{\tan x + 1}{1 - \tan x} + 1$$

$$\frac{\tan x + 1 + 1 - \tan x}{1 - \tan x}$$

*cosine right
side... b/c
sneaky!*

$$\frac{2}{1 - \tan x} \cdot \frac{\cos x}{\cos x}$$

$$\frac{2 \cos x}{\cos x - \cos x \tan x}$$

$$\frac{2 \cos x}{\cos x - \cos x \cdot \frac{\sin x}{\cos x}}$$

$$\frac{2 \cos x}{\cos x - \sin x} \cdot \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$\frac{\sqrt{2} \cos x}{\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x}$$

$$\frac{\sqrt{2} \cos x}{\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4}}$$

$$\frac{\sqrt{2} \cos x}{\cos(x + \frac{\pi}{4})}$$

$$\frac{\sqrt{2} \cos x}{\cos(x + \frac{\pi}{4})} \Rightarrow \sqrt{2} \cos x \cdot \sec(x + \frac{\pi}{4})$$

$$\cos \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$9. (\cos A \cdot \cos B - \sin A \cdot \sin B)^2 + (\sin A \cdot \cos B + \cos A \cdot \sin B)^2 = 1$$

Hint: Don't foil!

$$[\cos(A-B)]^2 + [\sin(A-B)]^2$$

$$\cos^2(A-B) + \sin^2(A-B)$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 \quad \text{Q.E.D.}$$

$$10. \sin \frac{3x}{7} \cdot \cos \frac{4x}{7} + \cos \frac{3x}{7} \cdot \sin \frac{4x}{7} = \sin x$$

$$\sin \left(\frac{3x}{7} + \frac{4x}{7} \right)$$

$$\sin \frac{7x}{7}$$

$$\sin x \text{ } \textcircled{smile}$$

$$11. \sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$$

$$\frac{1}{2} [(\cos x \cos y + \sin x \sin y) - (\cos x \cos y - \sin x \sin y)]$$

$$\frac{1}{2} (2 \sin x \sin y)$$

$$\sin x \sin y \text{ } \textcircled{smile}$$

$$12. \frac{\cos(x+y)}{\sin x \cdot \cos y} = \cot x - \tan y$$

$$\frac{\cos x \cos y - \sin x \sin y}{\sin x \cos y}$$

$$\frac{\cos x \cos y}{\sin x \cos y} - \frac{\sin x \sin y}{\sin x \cos y}$$

$$\cot x - \tan y \text{ } \textcircled{smile}$$