7.2 Operations with Polynomials

~~Essential Question:~~

What defines a polynomial and how do I perform operations using polynomials?

7.2 Operations with Polynomials

Essential Question: What defines a polynomial and how do I perform operations using polynomials?

A **term** is an algebraic expression that can be written using constants, variables, multiplication and division. The constants are called coefficients. A **polynomial** can be written using terms and addition and subtraction. The term of the polynomial which does not include a variable is called the constant term. Any letter may be used as the variable in a polynomial.

Note the characteristics of a polynomial.

- All exponents are whole numbers.
- No variables in the denominator.
- No variables under a radical.

Any letter may be used as the variable in a polynomial. Examples of **polynomials** include the following.

POLYNOMIALS	NOT POLYNOMIALS
$x^3-6x+\frac{1}{2}$ $\quad y-6.7$ $\quad 12$	3^x-7 $\quad \frac{1}{x^2}-2$ $\quad x^{\frac{1}{2}}-4$ $\quad \sqrt{x}-4$

7.2 Operations with Polynomials

Essential Question: What defines a polynomial and how do I perform operations using polynomials?

Degree of a Polynomial – The *exponent* of the highest power of x is the **degree** of the polynomial, and the coefficient of this highest power of the variable is the **leading coefficient.**

$x^0=1$

Polynomial	Degree	Leading Coefficient	Constant Term
$6x^7+4x^3+5x^2-7x+10$	7	6	10
$1x^3+0$	3	1	0
$12 \cdot x^0$	0	12	12
$2x^6+3x^7-1x^8-2x-4$	8	-1	-4

Polynomial functions of degree less that 5 are often referred to by special names.

- First-degree polynomial functions are called linear **functions.** $y=x+3$
- Second-degree polynomial functions are called quadratic **functions.** $y=x^2-x+2$
- Third- degree polynomial functions are called cubic **functions.**
- Fourth- degree polynomial functions are called quartic **functions.**

7.2 Operations with Polynomials

Essential Question: What defines a polynomial and how do I perform operations using polynomials?

Adding and Subtracting Polynomials

To add or subtract polynomials, combine like terms.

ex. $(-2x^3 + x^2 - 4x + 1) - (2x^3 + x - 4)$

$-4x^3 + x^2 - 3x - 3$

Multiplying Polynomials

To multiply polynomials, distribute/FOIL.

ex. $(2x - 3)(x^2 + 3x - 5)$

$2x^3 + 6x^2 - 10x - 3x^2 - 9x + 15$

$2x^3 + 3x^2 - 19x + 15$

7.2 Operations with Polynomials

Essential Question: What defines a polynomial and how do I perform operations using polynomials?

Dividing Polynomials

ex. $(3x^4 - 8x^2 - 11x + 1) \div (x - 2)$

$0x^3$

$$\begin{array}{r} 93\ 0\ R1 \\ 4\overline{)372} \\ -36 \\ \hline 12 \\ -12 \\ \hline 1 \end{array}$$

Synthetic Division

only works when divisor is first degree binomial

$$\begin{array}{r|rrrrr} 2 & 3 & 0 & -8 & -11 & 1 \\ & \downarrow & 6 & 12 & 8 & -6 \\ \hline & 3 & 6 & 4 & -3 & \boxed{-5} \end{array} \leftarrow R$$

$3x^3 + 6x^2 + 4x - 3$ R-5

Synthetic basics

1. Bring down 1st #
2. Multiply
3. Add down column
4. Repeat for each column
5. Answer is one degree less

Long Division

$$\begin{array}{r} 3x^3 + 6x^2 + 4x - 3 \text{ R-5} \\ x-2\overline{)3x^4 + 0x^3 - 8x^2 - 11x + 1} \\ -(3x^4 + 6x^3) \\ \hline 6x^3 - 8x^2 \\ -(6x^3 - 12x^2) \\ \hline 4x^2 - 11x \\ -(4x^2 - 8x) \\ \hline -3x + 1 \\ -(-3x + 6) \\ \hline -5 \end{array}$$

To check your answer...

answer * divisor + remainder = original