Name:_____

7.3 Factoring with Synthetic Division

1. Given that 3 is a zero of the polynomial $P(x) = x^3 + 2x^2 - 11x - 12$, factor completely

3 | 1 2 -11 -12
↓ 3 15 12
1 5 4 | 0

$(x-3)(x^2+5x+4)$

x^2+5x+4

$4 \times 1 = 4$, $4 + 1 = 5$

$(x+4)(x+1)$

$(x-3)(x+4)(x+1)$

2. Given that 2 is a zero of the polynomial $P(x) = x^3 - 4x^2 - 11x + 30$, factor completely

2 | 1 -4 -11 30
↓ 2 -4 -30
1 -2 -15 | 0

$(x-2)(x^2-2x-15)$

$x^2-2x-15$

$-5 \times 3 = -15$, $-5 + 3 = -2$

$(x-5)(x+3)$

$(x-2)(x-5)(x+3)$

List all possible rational zeros given by the Rational Zeros Theorem (but don't check to see which actually are zeros)

3. $P(x) = x^3 - 4x^2 + 3$

$p: \pm 1, \pm 3$

$q: \pm 1$

$\frac{p}{q}: \{\pm 1, \pm 3\}$

4. $P(x) = 6x^4 - x^2 + 2x + 12$

$p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$q: \pm 1, \pm 2, \pm 3, \pm 6$

$\frac{p}{q}: \pm\{1, 2, 3, 4, 6, 12, \frac{2}{3}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2}, \frac{4}{3}\}$

Factor Completely

5. $P(x) = x^3 + 3x^2 - 4$

$p: \pm 1, \pm 2, \pm 4$

$q: \pm 1$

$\frac{p}{q}: \pm\{1, 2, 4\}$

1 | 1 3 0 -4
↓ 1 4 4
1 4 4 | 0

$(x-1)(x^2+4x+4)$

x^2+4x+4

$2 \times 2 = 4$, $2 + 2 = 4$

$(x+2)(x+2)$

$(x-1)(x+2)^2$

6. $P(x)=x^3-3x-2$

P: ±1, ±2
Q: ±1
$\frac{P}{Q}$: ±{1, 2}

2 | 1 0 −3 −2
↓ 2 4 2
1 2 1 | 0

$(x-2)(x^2+2x+1)$

x^2+2y+1
1, 1, 1, 2
$(x+1)(x+1)$

$(x-2)(x+1)^2$

7. $P(x)=x^4-2x^3-5x^2+6x$

$P(x)=x(x^3-2x^2-5x+6)$

P: ±1, ±2, ±3, ±6
Q: ±1
$\frac{P}{Q}$: ±{1, 2, 3, 6}

1 | 1 −2 −5 6
↓ 1 −1 −6
1 −1 −6 | 0

$x(x-1)(x^2-x-6)$

x^2-x-6
−6, −3, −1, 2
$(x-3)(x+2)$

$x(x-1)(x-3)(x+2)$

Find all real zeros of the polynomial. Use the quadratic formula if necessary

8. $P(x)=x^3+4x^2+3x-2$

P: ±1, ±2
Q: ±1
$\frac{P}{Q}$: ±{1, 2}

−2 | 1 4 3 −2
↓ −2 −4 2
1 2 −1 | 0

$(x+2)(x^2+2x-1)$
↑prime (quadForm)

$x=\frac{-2\pm\sqrt{2^2-4(1)(-1)}}{2(1)}$
$=\frac{-2\pm\sqrt{8}}{2}$
$=\frac{-2\pm2\sqrt{2}}{2}$

$\{-2, -1\pm\sqrt{2}\}$

9. $P(x)=x^4-5x^3-5x^2+23x+10$

P: ±1, ±2, ±5, ±10
Q: ±1
$\frac{P}{Q}$: ±{1, 2, 5, 10}

−2 | 1 −5 −5 23 10
↓ −2 14 −18 −10
1 −7 9 5 | 0

$(x+2)(x^3-7x^2+9x+5)$

$x^3-7x+9x+5$
P: ±1, ±5
Q: ±1

5 | 1 −7 9 5
↓ 5 −10 −5
1 −2 −1 | 0

$(x-5)(x^2-2x-1)$
↑prime

$x=\frac{+2\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$
$=\frac{2\pm\sqrt{8}}{2}$

$\{-2, 5, 1\pm\sqrt{2}\}$