7.4 Sinusoidal Functions as Mathematical Models   Name:_____

1. Huckleberry Finn sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d from the water's surface was a sinusoidal function of time. When his stopwatch read 6 seconds, the point was at its highest 15 feet above the water's surface. The wheel's diameter was 18 feet (part of the wheel is always underwater), and it completed a revolution every 14 seconds.

　　a.) Sketch a graph of this sinusoid.

　　b.) Write an equation for this sinusoid.

　　c.) Where was the point when Huck started his stopwatch?

　　d.) Where was the point when Huck's stopwatch read 20 seconds?

　　e.) What is the first positive value of time at which the point was at the water's surface? At that time, was it going into or coming out of the water?

　　f.) When was the point 10 feet above the surface for the third time?

2. Researchers find a creature from an alien planet and discover that its body temperature varies sinusoidally with time. 35 minutes after they start timing, it reaches a high of 120° F. 20 minutes after that it reaches its next low, 104° F.

    a.) Sketch a graph of this sinusoid.

    b.) Write an equation expressing the alien's temperature in terms of minutes since the researchers starting timing.

    c.) What was its temperature when they started timing?

    d.) Find the first three times after they starting timing at which its temperature was 114° F.

3. The original Ferris wheel, built by George Ferris for the 1893 World's Fair, was much larger and slower than its modern counterparts. It had a diameter of 250 feet and contained 36 cars, each of which held 40 people. It made one revolution every 10 minutes and reached a maximum height of 264 feet. Grover Cleveland was given a private ride. He got on and the wheel starting slowly turning.

   a.) Sketch a graph of this sinusoid.

   b.) Write an equation expressing Grover's height above the ground in terms of time (in minutes) since the Ferris wheel started turning.

   c.) How high was Grover after 16 minutes?

   d.) When was he 200 feet above the ground for the 4th time?

4. You are on the beach in Galveston, Texas. At 2:00 pm on June 15th, the tide is in (the water level is at its deepest). At that time you find that the depth at the end of the pier is 1.5 meters. At 8:00 pm the same day when the tide is out, you find that the depth of the water is 1.1 meters. Assume that the depth of the water varies sinusoidally with time.

   a.) Write an equation expressing the depth of the water in terms of the number of hours that have elapsed since 12:00 noon on June 15th.

   b.) What will the depth of the water be at 4:00 pm on June 15th?

   c.) What will the depth of the water be at 7:00 am on June 16th?
       (First find how many hours have elapsed since 12 noon on June 15th.)

   d.) At what time will the first low tide occur on June 16th?

5. When a spaceship is fired into orbit from a site such as Cape Canaveral, which is not on the equator, it goes into an orbit that takes it alternatively north and south of the equator. It's distance from the equator is a sinusoidal function of time. Suppose that Discovery is launched from Cape Canaveral. 10 minutes after lift-off, it reaches its farthest distance north of the equator, 4000 kilometers. Half a cycle later it reaches its farthest distance south of the equator on the other side of the earth, also 4000 km. The Discovery completes an orbit every 90 minutes. Let y be the number of kilometers the space shuttle is north of the equator. (Consider distances south of the equator to be negative.) Let t be the number of minutes since lift-off.

    a.) Sketch a graph of this situation.

    b.) Write an equation expressing y in terms of t.

    c.) What is the space ship's distance from the equator when $t = 163$. Is the space shuttle north or south of the equator at this time?

    d.) Calculate the distance of Cape Canaveral from the equator by finding y when $t = 0$.

    e.) What is first time at which Discovery is 1600 km south of the equator?

6. You are on board a submarine, submerged in the Pacific Ocean. At time $t = 0$ you make contact with an enemy destroyer. Immediately you start porpoising (going deeper and shallower). At time $t = 4$ minutes you are at your deepest, 1000 meters **below** the surface. (Let the surface of the water be $y = 0$.) At time $t = 9$ minutes you are at your next shallowest point, 200 meters below the surface. Your depth varies sinusoidally with time.

    a.) Sketch a graph of your depth with respect to time.

    b.) Write an equation expressing your depth in terms of time.

    c.) Your submarine is safe from the enemy destroyer when it is 300 meters or more below the surface. Was your submarine safe at time $t = 0$? Justify your answer.

    d.) Between what two times is your submarine first safe?

7. As you stop your car at a traffic light, a pebble becomes wedged between the tire treads. When you start off, the distance of the pebble from the pavement varies sinusoidally with the distance you have traveled. The period is the circumference of the wheel, and the diameter of the wheel is 24 inches.

    a.) Sketch a graph of this function.

    b.) Write an equation of this function.

    c.) Predict the pebble's distance from the pavement when you have driven 35 inches.

    d.) What are the first two distances you have traveled when the pebble is 11 inches from the pavement?