7.8 Notes: Operations and Compositions of Functions

EQ: How do I combine functions?

Operations with Functions

For two functions $f(x)$ and $g(x)$:

$$(f+g)(x)=f(x)+g(x),\quad (f-g)(x)=f(x)-g(x),\quad (fg)(x)=f(x)\cdot g(x),\quad \left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\ g(x)\neq 0$$

Example 1: If $f(x)=3x-2$ and $g(x)=x^2-5$, find the following. State the domain of each.

a) $(f-g)(x)$

$3x-2-(x^2-5)$

$3x-2-x^2+5$

$-x^2+3x+3$

b) $(fg)(x)$

$(3x-2)(x^2-5)$

$3x^3-15x-2x^2+10$

Composition of Functions For two functions $f(x)$ and $g(x)$: $g\circ f(x)=g(f(x))$

Example 2: If $f(x)=x+4$ and $g(x)=\sqrt{x}$, find the following. State the domain in e) and f).

a) $f(g(3))$

$g(3)=\sqrt{3}$

$f(\sqrt{3})=\sqrt{3}+4$

b) $g(f(-2))$

$f(-2)=-2+4=2$

$g(2)=\sqrt{2}$

c) $f(g(9))$

$g(9)=\sqrt{9}=3$

$f(3)=3+4=7$

d) $g(f(-5))$

$f(-5)=-5+4=-1$

$g(-1)=\sqrt{-1}=i$

or DNE

e) $f(g(x))$

$y=\sqrt{x}+4$

f) $g(f(x))$

$\sqrt{x+4}$

Example 3: If $f(x)=x^2-4$ and $g(x)=x+3$, find the following:

a) $f(g(-5))$

$g(-5)=-5+3=-2$

$f(-2)=(-2)^2-4$

$4-4=0$

b) $f(g(x))$

$(x+3)^2-4$

b) $g(f(x))$

x^2-4+3

x^2-1