

Pre AP Pre Calculus
5.7 Double Angle Properties

Name: _____
Date: _____ Period : _____

Find the exact values for $\sin 2x$, $\cos 2x$, and $\tan 2x$ under the given conditions:

1. $\sin x = -\frac{4}{5}$ and $\frac{3\pi}{2} < x < 2\pi$

$-\frac{24}{25}, -\frac{7}{25}, \frac{24}{7}$

2. $\sec x = -5$ and $\pi < x < \frac{3\pi}{2}$

$\frac{4\sqrt{6}}{25}, -\frac{23}{25}, -\frac{4\sqrt{6}}{23}$

For each equation, find:

- a) the general solution b) the particular values for $0 \leq x < 2\pi$ or $0 \leq \theta < 360^\circ$

3. $4\sin x \cos x = \sqrt{3}$

A. $\frac{\pi}{6} + \pi n = x$ B. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{3\pi}{6}, \frac{4\pi}{3}$

$\frac{\pi}{3} + \pi n = x$

5. $\cos^2 \theta - \sin^2 \theta = -1$

A. $\frac{\pi}{2} + \pi n = x$ B. $\frac{\pi}{2}, \frac{3\pi}{2}$

7. $\frac{2\tan x}{1 - \tan^2 x} = \sqrt{3}$ B. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{3\pi}{6}, \frac{5\pi}{3}$

$\frac{\pi}{6} + \frac{\pi}{2}n$

4. $4\sin x \cos x = -\sqrt{2}$

A. $\frac{3\pi}{8} + \pi n$ B. $\frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

$\frac{5\pi}{8} + \pi n$

6. $1 - 2\sin^2 x = -\frac{1}{2}$

A. $\frac{\pi}{3} + \pi n$ B. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\frac{2\pi}{3} + \pi n$

8. $\frac{2\tan \theta}{1 - \tan^2 \theta} = -1$

B. $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

$\frac{3\pi}{8} + \frac{\pi}{2}n$

Using the double angle properties, write an equation expressing:

9. $\tan 18x$ in terms of $\tan 9x$ $\frac{2\tan(9x)}{1 - \tan^2(9x)}$

10. $\cot 14x$ in terms of $\tan 7x$ $\frac{1 - \tan^2(7x)}{2\tan(7x)}$

11. $\cos 10x$ in terms of $\cos 5x$ and $\sin 5x$

$\cos^2(5x) - \sin^2(5x)$

12. $\sin 6x$ in terms of $\cos 3x$ and $\sin 3x$

$2\sin(3x)\cos(3x)$

13. $\cos 6x$ in terms of $\cos 3x$

$2\cos^2(3x) - 1$

14. $\cos 22x$ in terms of $\sin 11x$

$1 - 2\sin^2(11x)$

Simplify each expression using the double angle properties:

15. $\frac{\sin 2x}{2\sin x} = \cos x$

16. $2\cos 2y \cdot \sin 2y = \sin(4y)$

17. $1 - 2\sin^2 3k = \cos(6k)$

18. $\frac{2\tan\left(\frac{1}{2}x\right)}{1 - \tan^2\left(\frac{1}{2}x\right)} = \tan x$

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Prove each identity (Pick any 5):

19.
$$\frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$$

21.
$$\tan x = \frac{1 - \cos 2x}{\sin 2x}$$

22.
$$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

23.
$$\sin 2x = 2 \cot x \sin^2 x$$

24.
$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

25.
$$\frac{\sin 2x}{1 + \cos 2x} = \tan x$$

26.
$$\frac{1}{2}(1 + \cos 2x) = \cos^2 x$$

27.
$$(1 + \tan x) \tan 2x = \frac{2 \tan x}{1 - \tan x}$$

28.
$$\cos^4 x - \sin^4 x = \cos 2x$$

29.
$$\sec 2x = \frac{1}{1 - 2 \sin^2 x}$$

30.
$$\frac{1 + \cos 2x}{\sin 2x} = \cot x$$

$$\textcircled{1} \quad \sin x = -\frac{4}{5} \quad \frac{3\pi}{2} < x < 2\pi \quad QIV$$



$$\textcircled{2} \quad \sec x = -5 \quad \pi < x < \frac{3\pi}{2} \quad QIII$$



$$\textcircled{3} \quad \frac{4 \sin x \cos x}{2} = \frac{\sqrt{13}}{2}$$

$$2 \sin x \cos x = \frac{\sqrt{13}}{2}$$

$$\sin 2x = \frac{\sqrt{13}}{2}$$

$$\sin^{-1}\left(\frac{\sqrt{13}}{2}\right) = 2x$$



$$\frac{2\pi}{3} + 2\pi n = 2x$$

General Solution $\boxed{\frac{\pi}{6} + \pi n = x}$

B. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

$$\textcircled{5} \quad \cos^2 \theta - \sin^2 \theta = -1$$

$$\cos 2x = -1$$

$$\cos^{-1}(-1) = 2x$$

$$\pi + 2\pi n = 2x$$

$\boxed{\frac{\pi}{2} + \pi n = x}$

B. $\frac{\pi}{2}, \frac{3\pi}{2}$

$$\bullet \sin 2x = 2 \sin x \cos x = 2 \left(-\frac{4}{5}\right) \left(\frac{3}{5}\right) = \boxed{-\frac{24}{25}}$$

$$\bullet \cos 2x = \cos^2 x - \sin^2 x = \left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = \boxed{-\frac{7}{25}}$$

$$\bullet \tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-\frac{24}{25}}{-\frac{7}{25}} = \frac{24}{25} \cdot \frac{25}{7} = \boxed{+\frac{24}{7}}$$

$$\bullet \sin 2x = 2 \left(-\frac{2\sqrt{6}}{5}\right) \left(-\frac{1}{5}\right) = \boxed{\frac{4\sqrt{6}}{25}}$$

$$\bullet \cos 2x = \left(-\frac{1}{5}\right)^2 - \left(-\frac{2\sqrt{6}}{5}\right)^2 = \frac{1}{25} - \frac{24}{25} = \boxed{-\frac{23}{25}}$$

$$\bullet \tan 2x = \frac{4\sqrt{6}}{25} \cdot \frac{-25}{23} = \boxed{-\frac{4\sqrt{6}}{23}}$$

$$\textcircled{4} \quad \frac{4 \sin x \cos x}{2} = -\frac{\sqrt{12}}{2}$$

$$2 \sin x \cos x = -\frac{\sqrt{12}}{2}$$

$$\sin 2x = -\frac{\sqrt{12}}{2}$$

$$\sin^{-1}\left(-\frac{\sqrt{12}}{2}\right) = x$$



① $\frac{7\pi}{4} + 2\pi n = 2x$

③ $\frac{5\pi}{4} + 2\pi n = 2x$

B. $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

$\frac{5\pi}{8} + \pi n = x$

$$\textcircled{6} \quad 1 - 2 \sin^2 x = -\frac{1}{2}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = 2x$$



QIII $\frac{2\pi}{3} + 2\pi n = 2x$

QIII $\frac{4\pi}{3} + 2\pi n = 2x$

$\boxed{\frac{\pi}{3} + \pi n = x}$

B. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\boxed{\frac{2\pi}{3} + \pi n = x}$

$$\textcircled{7} \quad \frac{2\tan x}{1-\tan^2 x} = \sqrt{3}$$

$$\tan 2x = \sqrt{3}$$

$$\tan^{-1}(\sqrt{3}) = 2x$$

$$\frac{2}{\sqrt{3}} + \pi n = 2x$$

$$\frac{\pi}{6} + \frac{\pi}{2}n = x$$

B. $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

$$\textcircled{8} \quad \frac{2\tan \theta}{1-\tan^2 \theta} = -1$$

$$\tan 2\theta = -1$$

$$\tan^{-1}(-1) = 2\theta$$

$$\frac{7\pi}{4} + \pi n = 2\theta$$

$$\frac{3\pi}{8} + \frac{\pi}{2}n = x$$

B. $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

$$\textcircled{9} \quad \tan 18x = \tan(2 \cdot 9x)$$

$$\frac{2+\tan 9x}{1-\tan^2(9x)}$$

$$\textcircled{11} \quad \cos 10x = \cos(2 \cdot 5x)$$

$$\cos^2(5x) - \sin^2(5x)$$

$$\textcircled{13} \quad \cos 6x = \cos(2 \cdot 3x)$$

$$2\cos^2(3x) - 1$$

$$\textcircled{15} \quad \frac{\sin 2x}{2\sin x} = \frac{2\sin x \cos x}{2\sin x}$$

$$\textcircled{17} \quad 1 - 2\sin^2 3k$$

$$3k = x \quad \frac{\cos(2 \cdot 3k)}{\cos(6k)}$$

$$\textcircled{19} \quad \frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$$

$$\frac{\cos 2x (\cos x + \sin x)}{\cos^2 x - \sin^2 x}$$

$$\frac{\cos 2x (\cos x + \sin x)}{\cos 2x} \textcircled{1}$$

$$\textcircled{10} \quad \cot 14x = \frac{1}{\tan 2 \cdot 7x}$$

$$\frac{1-\tan^2(7x)}{2\tan(7x)}$$

$$\textcircled{12} \quad \sin 6x = \sin(2 \cdot 3x)$$

$$2 \sin(3x) \cos(3x)$$

$$\textcircled{14} \quad \cos 22x = \cos(2 \cdot 11x)$$

$$1 - 2\sin^2(11x)$$

$$\textcircled{16} \quad 2\cos 2y \cdot \sin 2y$$

$$\frac{\sin(2 \cdot 2y)}{\sin(4y)}$$

$$\sin 2x = 2\sin x \cos x$$

$$2y = x$$

$$\textcircled{18} \quad \frac{2\tan(\frac{1}{2}x)}{1-\tan^2(\frac{1}{2}x)} \quad x \rightarrow \frac{1}{2}x$$

$$\tan(2 \cdot \frac{1}{2}x) = \tan x$$

$$\textcircled{21} \quad \tan x = \frac{\frac{1-\cos 2x}{\sin 2x}}{\frac{1-(1-2\sin^2 x)}{2\sin x \cos x}}$$

$$\frac{2\sin^2 x}{2\sin x \cos x}$$

$$\frac{\sin x}{\cos x}$$

$$\tan x \textcircled{1}$$

$$②2 \cos^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}}$$

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\therefore \frac{\cos^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1}$$

$$②4 \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\frac{1}{2}(1 - (1 - 2\sin^2 x))$$

$$\frac{1}{2}(2\sin^2 x)$$

$$\sin^2 x \quad \text{☺}$$

$$②6 \frac{1}{2}(1 + \cos 2x) = \cos^2 x$$

$$\frac{1}{2}(1 + (2\cos^2 x - 1))$$

$$\frac{1}{2}(2\cos^2 x) \quad \text{☺}$$

$$②8 \cos^4 x - \sin^4 x = \cos 2x$$

$$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$$

$$\cos^2 x - \sin^2 x$$

$$\cos 2x \quad \text{☺}$$

$$③0 \frac{1 + \cos 2x}{\sin 2x} = \cot x$$

$$\frac{1 + 2\cos^2 x - 1}{2\sin x \cos x} \Rightarrow$$

$$\frac{x \cos^2 x}{2\sin x \cos x} \Rightarrow \frac{\cos x}{\sin x} \quad \text{☺}$$

$$②3 \sin 2x = 2 \cot \cdot \sin^2 x$$

$$2 \frac{\cos x}{\sin x} \cdot \sin^2 x$$

$$2 \cos x \sin x \\ \sin 2x \quad \text{☺}$$

$$②5 \frac{\sin 2x}{1 + \cos 2x} = \tan x$$

$$\frac{2\sin x \cos x}{1 + (2\cos^2 x - 1)}$$

$$\frac{2\sin x \cos x}{2\cos^2 x}$$

$$\frac{\sin x}{\cos x} \quad \text{☺}$$

$$②7 (1 + \tan x) \tan 2x = \frac{2 + \tan x}{1 - \tan x}$$

$$(1 + \tan x) \left(\frac{2 + \tan x}{1 - \tan x} \right)$$

$$(1 + \tan x) \left(\frac{2 + \tan x}{(1 - \tan x)(1 + \tan x)} \right)$$

$$\frac{2 + \tan x}{1 - \tan x} \quad \text{☺}$$

$$②9 \sec 2x = \frac{1}{1 - 2\sin^2 x}$$

$$\frac{1}{\cos 2x}$$

$$\sec 2x \quad \text{☺}$$