9.3 Slant Asymptotes

Warm-Up Thursday

Find all asymptotes and any removable discontinuities of $f(x) = \dfrac{x+6}{x^2 + 3x - 18}$

$$f(x) = \frac{\cancel{x+6}}{(x-3)\cancel{(x+6)}}$$

$$\frac{1}{x-3}$$

RD: $x + 6 = 0$

$x = -6$

$\dfrac{1}{-6-3}$

$\left(-6, -\frac{1}{9} \right)$

VA: $x = 3$

HA: $y = 0$

About Me

1. Who is your hero? Why?

2. Would you rather be kept away from your friends for 6 months or the internet for 2 years?
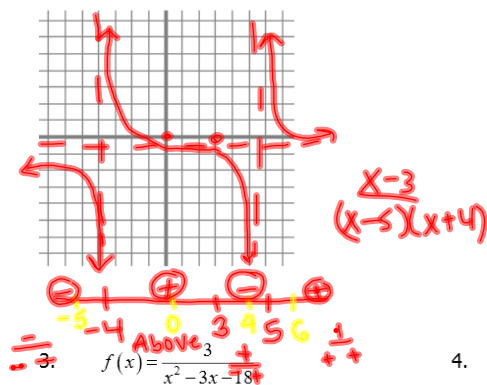
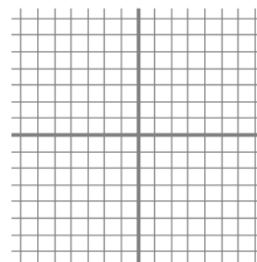9.2 Graphing Rational Functions Day 2 (Holes)   Name _____

Find the domain, vertical or horizontal asymptotes, removable discontinuities, and graph the function with their asymptotes.

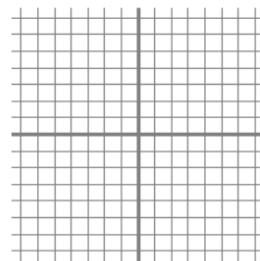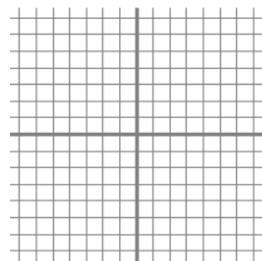1. $f(x) = \dfrac{x-3}{x^2-x-20}$ = $\dfrac{X-3}{(X-5)(X+4)}$

xint: (3,0)
y-int: (0, +3/20)

VA: X=5
      X=-4
HA: Y=0

2. $f(x) = \dfrac{x^2-9}{x-3}$

$\dfrac{X-3}{(X-5)(X+4)}$

⊖ ⊕ ⊖ ⊕
-5 -4 0 3 4 5 6
   Above 3
   $\frac{+}{+}$ $\frac{-}{+}$ $\frac{+}{+}$
$\frac{-}{-}$

3. $f(x) = \dfrac{x+3}{x^2-3x-18}$
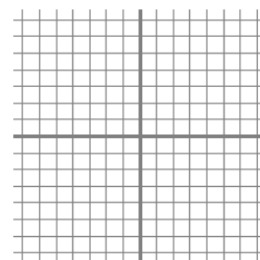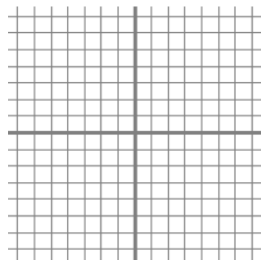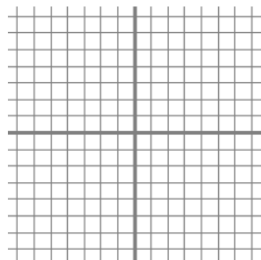
4. $f(x) = \dfrac{(x-2)(x+4)}{(x+8)(x-2)}$

5. $f(x) = \dfrac{x-4}{x^2+2x-24}$

6. $f(x) = \dfrac{2x^2+5x-3}{x^2+x-20}$

7. $f(x) = \dfrac{x^2+2x-3}{x^2+6x+9}$

9.3 Slant Asymptotes

EQ: How do I find slant asymptotes?

A Slant Asymptote (___oblique asymptote___) occurs on the graph of a rational function when... there is no HA ($\frac{high}{low}$) <u>AND</u> the degree of top is exactly <u>one</u> bigger
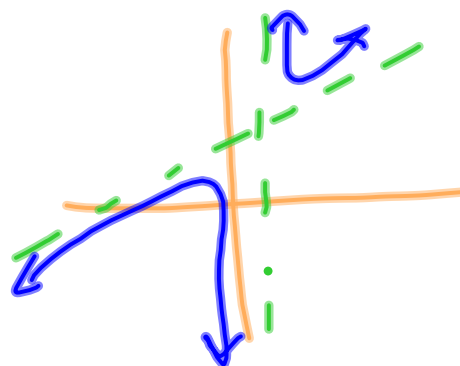
<u>Steps to find SA:</u>

1. Find HA

   (degree of top over degree of bottom)

2. high over low <u>and</u> exactly one degree higher on top

3. Use long division or synthetic division

4. Remainder doesn't matter!

4

9.3 Slant Asymptotes

EQ: How do I find slant asymptotes?

Graph each rational function and list all asymptotes.

EX1. $\dfrac{x^2 - 4x - 5}{x - 3}$ $= \dfrac{(x-5)(x+1)}{x-3}$ NO R.D.

VA: x = 3 ← synthetic

HA: none

  ↳ SA? Y = X − 1

x-int: (5, 0) & (−1, 0)

y-int: (0, 5/3)

$$+3\,\overline{\big)\ 1\ -4\ -5}$$
$$\ \downarrow\ \ 3\ -3$$
$$\ 1\ -1\ -8$$

Y = X − 1

Steps to find SA:

1. Find HA

    (degree of top over degree of bottom)

2. high over low <u>and</u> exactly one degree higher on top

3. Use long division or synthetic division

4. Remainder doesn't matter!

9.3 Slant Asymptotes

EQ: How do I find slant asymptotes?

EX2. Find all asymptotes of $f(x) = \dfrac{2x^3 - 2x^2}{x^2 - x - 30}$ **3**

2

$$f(x) = \frac{2x^2(x-1)}{(x-6)(x+5)}$$

VA: x = -5
x = 6

HA: none

↳ SA? yes

$$
\begin{array}{r}
2x + 0 \\
x^2 - x - 30 \overline{\smash)2x^3 - 2x^2 + 0x + 0} \\
-2x^3 + 2x^2 + 60x \downarrow \\
\hline
0x^2 + 60x + 0
\end{array}
$$

y = 2x

Steps to find SA:

1. Find HA

(degree of top over degree of bottom)

2. high over low and exactly one degree higher on top
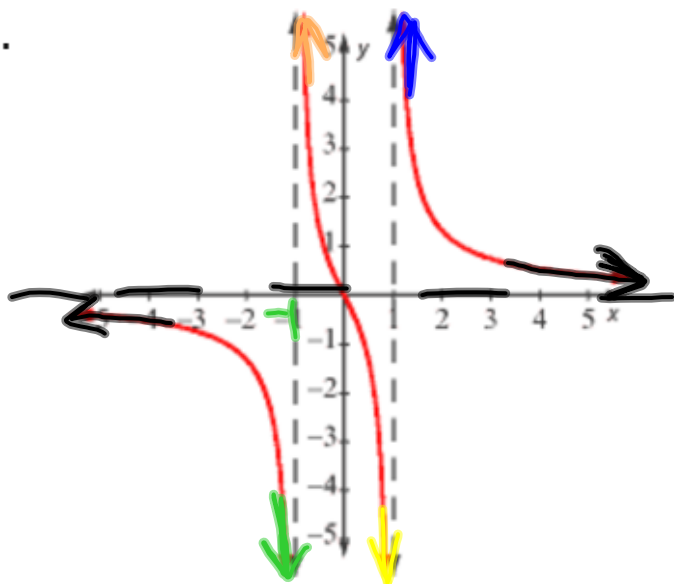
3. Use long division or synthetic division

4. Remainder doesn't matter!

9.3 Slant Asymptotes

EQ: How do I find slant asymptotes?

EX3.

Use the graph to determine:

as $x \to -1^+$, $f(x) \to$ _____ ∞

Right

as $x \to -1^-$, $f(x) \to$ _____ $-\infty$

Left

as $x \to +1^+$, $f(x) \to$ _____ ∞

as $x \to +1^-$, $f(x) \to$ _____ $-\infty$

HA
(end behavior)

as $x \to \infty$, $f(x) \to$ _____ 0

as $x \to -\infty$, $f(x) \to$ _____ 0

9.3 Slant Asymptotes

Rational Functions
Polynomial
Polynomial

Step 1: FACTOR

Step 2: Look for any factors that cancel

If a factor cancels out, it creates a _removable_ _discontinuity_ or _hole_

x-coordinate: _set canceled factor = 0_

y-coordinate: _plug x into remaining function_

After this point, just use the _rem._ _function_

Step 3: Find the vertical asymptote(s)

Set the *bottom* of the fraction = 0

Step 4: Find the horizontal asymptote

Bottom heavy degree: y = 0   _low high_

Equal degree: y = LC/LC   _same same_

Top heavy degree: No HA, maybe slant   _high low_

Top *1 higher degree* than bottom: SA, divide, ignore r

Step 5: Find the x-intercept(s)

Set the *top* of the fraction = 0

Step 6: Find the y-intercept

Plug in 0 for x

8

9.3 Slant Asymptotes

EQ: How do I find slant asymptotes?

Exit Ticket
on google classroom

HW Calendar (4 stamps)
due tomorrow end of class

9.3 Graphing Rational Functions (Slant Asymptotes)   Name _____

Use algebra to determine the location of the vertical asymptotes, horizontal asymptotes / slant asymptotes and any holes in the graph of the function. Then sketch the graph.

1. $f(x) = \dfrac{x+1}{3x^2 + x - 2}$

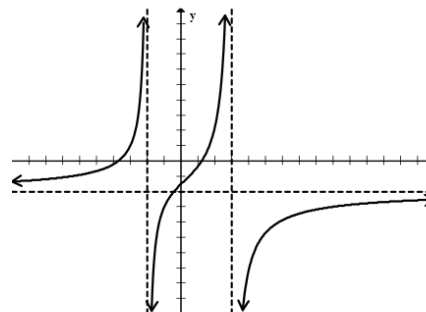2. $f(x) = \dfrac{x^2 - 2x - 8}{x + 3}$

3. $f(x) = \dfrac{x^2 + 3x - 4}{x - 2}$
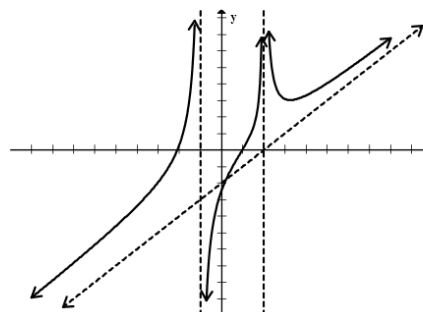
4. $f(x) = \dfrac{x + 6}{x^2 + 3x - 18}$

5. $f(x) = \dfrac{x^2 - x - 20}{x - 5}$

6. $f(x) = \dfrac{x^3 + 27}{x^2 - 9}$

11. Use the graph to determine:

as $x \to -2^+$, $f(x) \to$ _____

as $x \to -2^-$, $f(x) \to$ _____

as $x \to 3^+$, $f(x) \to$ _____

as $x \to 3^-$, $f(x) \to$ _____

as $x \to \infty$, $f(x) \to$ _____

as $x \to -\infty$, $f(x) \to$ _____

12. Use the graph to determine:

as $x \to -1^+$, $f(x) \to$ _____

as $x \to -1^-$, $f(x) \to$ _____

as $x \to 2^+$, $f(x) \to$ _____

as $x \to 2^-$, $f(x) \to$ _____

as $x \to \infty$, $f(x) \to$ _____

as $x \to -\infty$, $f(x) \to$ _____

For #13-14, Write the equation of the function with the given characteristics.

13. hole at $x = -5$
VA at $x = 2$ and $x = -4$
x-intercepts at $x = -6$ and $x = 3$
HA at $y = 3$

14. VA at $x = 1$ and $x = 3$
has a slant asymptote
only x-intercepts are $x = 2$ and $x = 5$