

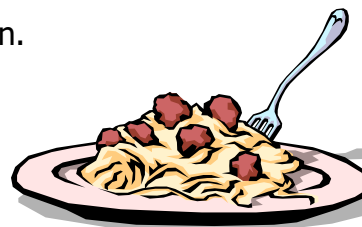
Introduction to Sine and Cosine Graphs

Spaghetti Trig

Materials:

White computer paper – 1 piece for each student as they walk in.
For each pair:

Tape - one 5" piece
Yarn about 1 foot
Glue Stick
Spaghetti – 10 to 12 noodles
Marker (a different color than the yarn)
Packet with Procedure and Unit Circle (attached)



Procedure:

1. Half of the class will be making a COSINE graph. Use X coordinates only!
The other half of you are making a SINE graph. Use Y coordinates only!
2. Find a partner and tape your pieces of paper together small end to small end.
3. On the copy of your unit circle in this handout, label the exact values for all the special (30, 45 and 60 degree) angles in each quadrant. Mark the ordered pairs for the points where the unit circle intersects the x and y axes as well (quadrantal angles 0, 90, 180, 270).
4. Wrap the string around the unit circle. One end of the string should be placed at the point (1 , 0). Use a marker to mark points on the string at each of the special and quadrantal angles.

We are interested in finding a function in terms of the angle θ that will give us the value of the x-coordinate of each point and a function in terms of the angle θ that will give us the value of the y-coordinate of each point.

Reminder $x = \cos \theta$ and $y = \sin \theta$. Let $g(\theta) = \cos \theta$ and $f(\theta) = \sin \theta$. This means that if we are to graph the sine or cosine function, not both on your two sheets of paper, and our new independent variable is the angle θ .

5. Draw a set of coordinate axes on one of the taped pairs of paper. Place the paper horizontally and place the origin in the center of the paper. Since θ is the independent variable, the horizontal axis should be labeled θ and the vertical axis should be labeled $g(\theta) = \cos \theta$ or $f(\theta) = \sin \theta$.
6. The yarn can now be placed along the horizontal axis with the end of the string, that started at the point (1 , 0) on your unit circle, now placed at the origin since this represents $\theta = 0$. The marks that we made on the yarn will give us the angle measures that we need. Thus the first mark can be transferred to the paper and labeled $\frac{\pi}{6}$. The second mark can be transferred to the paper and labeled $\frac{\pi}{4}$, etc. Once you have labeled the positive horizontal axis, flip the string over the y axis and label the negative horizontal axis using the same marks. Make sure you use negative angle measures to the left of the y axis.

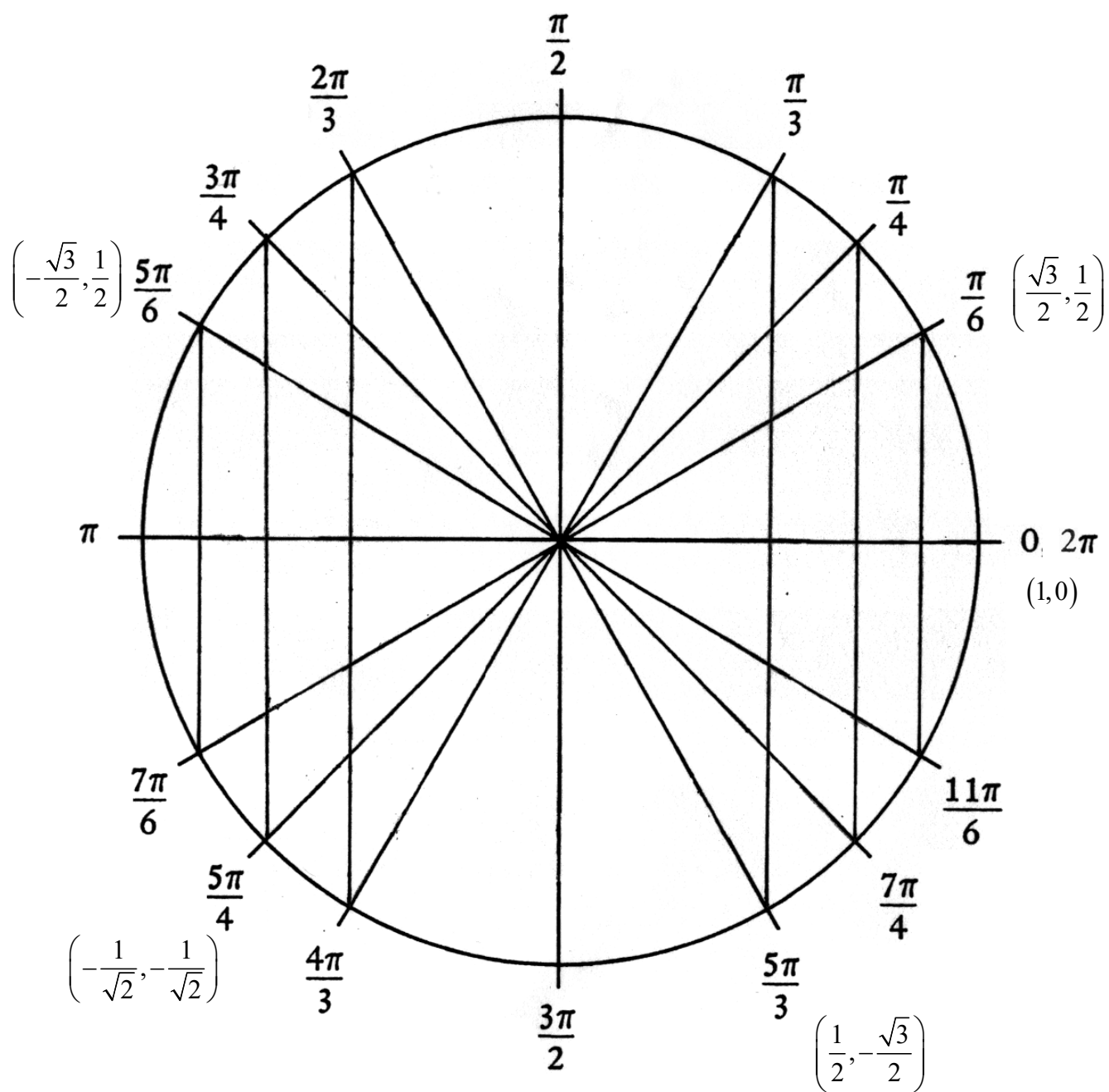
7. Go back to the unit circle. Since we are interested in the sine function (which corresponds to the y-value of each point) or the cosine function (which corresponds to the x-value of each point), we are interested in the distance a given point is from the horizontal or vertical axis. This is where the spaghetti will help.

For Sine Graphs: at an angle measure of 0, the y value is 0, thus we will not need any spaghetti. However, at $\frac{\pi}{6}$, the y value is $\frac{1}{2}$. Place a noodle of spaghetti vertically with one end on the horizontal axis and the remainder of the noodle going through the point $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Break the noodle off at that point. The noodle gives us a visual image of (the y-value) $\sin \theta$ at $\frac{\pi}{6}$. Take that piece of spaghetti and glue (or place) it on the paper at $x = \frac{\pi}{6}$. Since the y-value was positive, then the spaghetti should be placed **above** the horizontal axis.

For Cosine Graphs: at an angle measure of 0, the x value is 1. Place a noodle of spaghetti horizontally with one end on the vertical axis and the remainder of the noodle going through the point (1 , 0). Break the noodle off at that point. The noodle gives us a visual image of (the x-value) at 0. Take that piece of spaghetti and glue (or place) it on the paper at 0. Since the x-value was positive, then the spaghetti should be placed **above** the horizontal axis.

For All Graphs: Repeat this process for all the special and quadrantal angles. When you get to the third and fourth quadrants, the y-values are now negative. Therefore, you have to place the spaghetti **below** the horizontal axis.

8. Put your names on the front of your project and hang it in the hallway.



Name: _____

Amplitude and Period

Sketch two complete cycles of each graph.
Label all critical points.

1. $y = 3 \sin 5\theta$ (degrees)

2. $y = 12 \cos \frac{3}{4}\theta$ (degrees)

3. $y = 3 \sin \frac{\pi}{4}x$ (radians)

4. $y = 7 \cos 2x$ (radians)

5. $y = 20 \cos \frac{1}{3}x$ (radians)

6. $y = 5 \sin 4\pi\theta$ (radians)

7. $y = 8 \sin \frac{\pi}{5}x$ (radians)

8. $y = 10 \cos \frac{1}{6}x$ (radians)

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2.2 General Sinusoidal Graphs

Name: _____

Sketch two complete cycles of each graph.
Label all critical points.

State the domain and range of each graph.

1. $y = 7 + 4 \cos 3(\theta - 10^\circ)$ (degrees)

2. $y = -3 + 5 \sin(\theta + 200^\circ)$ (degrees)

3. $y = 2 + 6 \sin \frac{\pi}{4}(x - 1)$ (radians)

4. $y = 23 + 30 \cos \frac{1}{5}(x - \pi)$ (radians)

5. $y = 5 \cos\left(3x - \frac{\pi}{4}\right)$ (radians)

6. $y = 15 - 2 \cos \frac{3}{2}\left(\theta + \frac{\pi}{2}\right)$ (radians)

7. $y = 8 - \sin(2x + \pi)$ (radians)

8. $y = -2 + 5 \cos \frac{\pi}{15}(x + 5)$ (radians)

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Notes:

$$y = -3 + 5 \cos \frac{2}{3}(\theta + 150)$$

$$y = 5 - 6 \cos \frac{\pi}{5}(x - 2)$$

$$y = -\sin(3x + \frac{\pi}{2}) + 1$$

General Sinusoidal Graphs Key

Sketch two complete cycles of each graph. Label all critical points.
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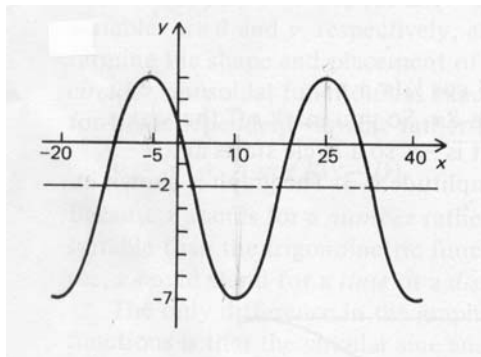
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2.3 Writing Equations from Graphs

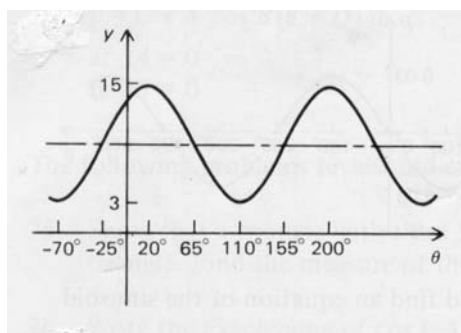
Name: _____

Given the graph write an equation in either sine or cosine. Check the axis for degrees or radians

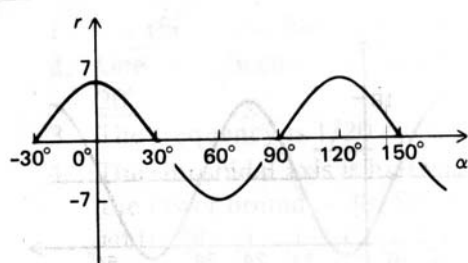
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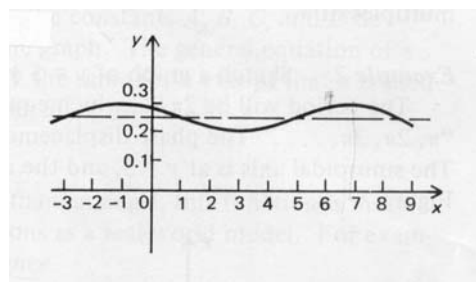
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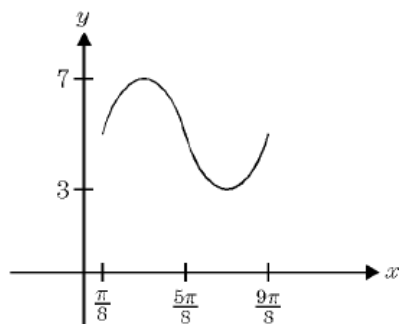
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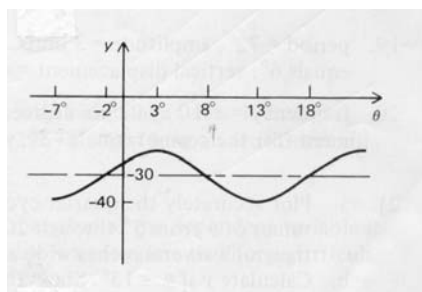
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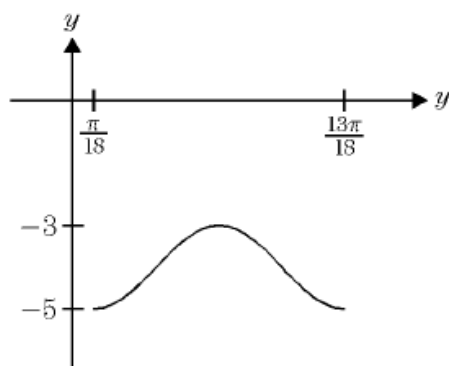
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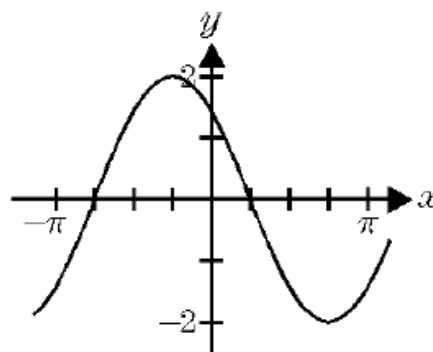
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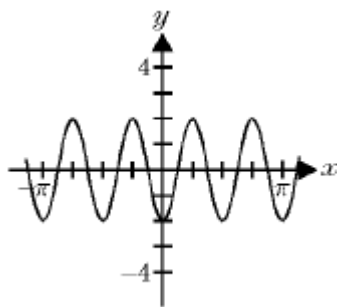
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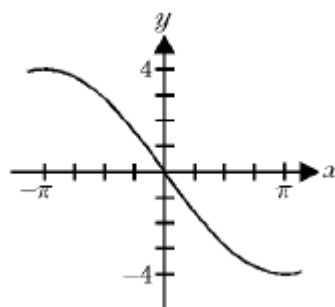
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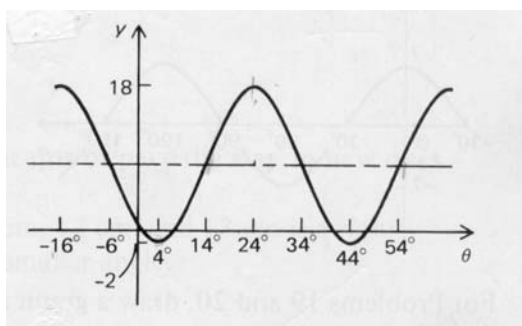
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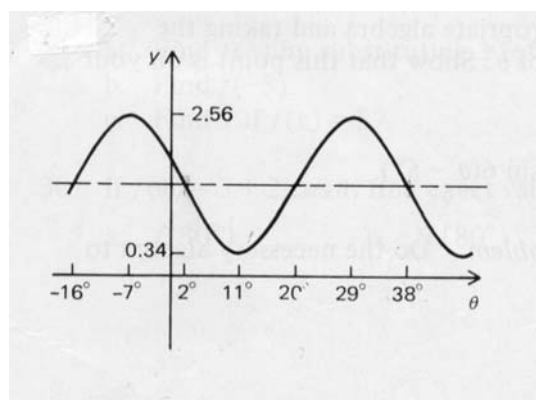
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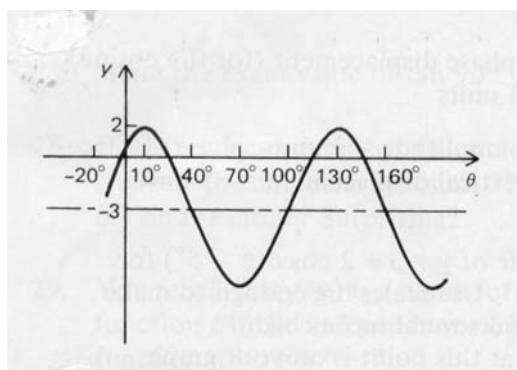
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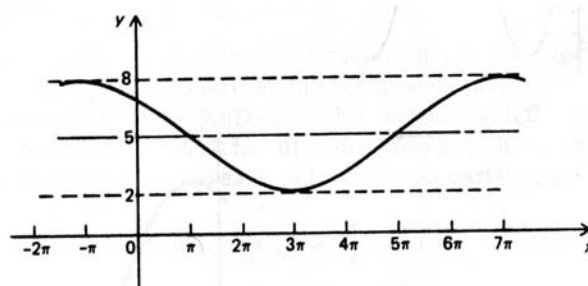
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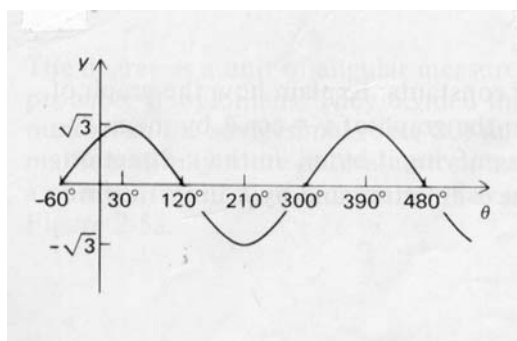
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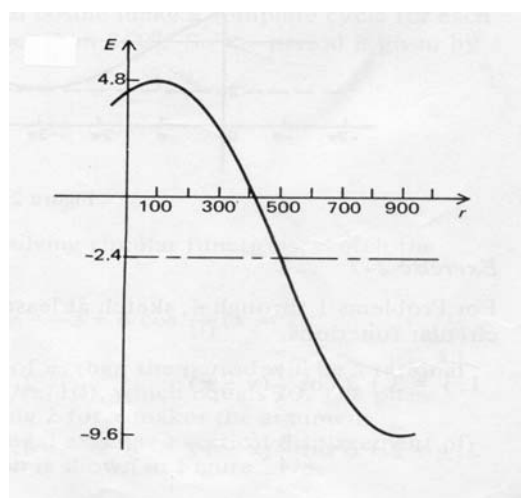
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15.



16.



2.4 Sinusoidal Functions as Mathematical Models (1) Name: _____

1. Huckleberry Finn sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d from the water's surface was a sinusoidal function of time. When his stopwatch read 6 seconds, the point was at its highest 15 feet above the water's surface. The wheel's diameter was 18 feet (part of the wheel is always underwater), and it completed a revolution every 14 seconds.

a.) Sketch a graph of this sinusoid.

b.) Write an equation for this sinusoid.

c.) Where was the point when Huck started his stopwatch?

d.) Where was the point when Huck's stopwatch read 20 seconds?

e.) What is the first positive value of time at which the point was at the water's surface? At that time, was it going into or coming out of the water?

f.) When was the point 10 feet above the surface for the third time?

2. Researchers find a creature from an alien planet and discover that its body temperature varies sinusoidally with time. 35 minutes after they start timing, it reaches a high of 120°F . 20 minutes after that it reaches its next low, 104°F .
- Sketch a graph of this sinusoid.
 - Write an equation expressing the alien's temperature in terms of minutes since the researchers starting timing.
 - What was its temperature when they started timing?
 - Find the first three times after they starting timing at which its temperature was 114°F .

3. The original Ferris wheel, built by George Ferris for the 1893 World's Fair, was much larger and slower than its modern counterparts. It had a diameter of 250 feet and contained 36 cars, each of which held 40 people. It made one revolution every 10 minutes and reached a maximum height of 264 feet. Grover Cleveland was given a private ride. He got on and the wheel starting slowly turning.

a.) Sketch a graph of this sinusoid.

b.) Write an equation expressing Grover's height above the ground in terms of time (in minutes) since the Ferris wheel started turning.

c.) How high was Grover after 16 minutes?

d.) When was he 200 feet above the ground for the 4th time?

2.5 Sinusoidal Functions as Mathematical Models (2) Name: _____

1. You are on the beach in Galveston, Texas. At 2:00 pm on June 15th, the tide is in (the water level is at its deepest). At that time you find that the depth at the end of the pier is 1.5 meters. At 8:00 pm the same day when the tide is out, you find that the depth of the water is 1.1 meters. Assume that the depth of the water varies sinusoidally with time.
 - a. Write an equation expressing the depth of the water in terms of the number of hours that have elapsed since 12:00 noon on June 15th.
 - b. What will the depth of the water be at 4:00 pm on June 15th?
 - c. What will the depth of the water be at 7:00 am on June 16th?
(First find how many hours have elapsed since 12 noon on June 15th.)
 - d. At what time will the first low tide occur on June 16th?

2. When a spaceship is fired into orbit from a site such as Cape Canaveral, which is not on the equator, it goes into an orbit that takes it alternatively north and south of the equator. It's distance from the equator is a sinusoidal function of time. Suppose that Discovery is launched from Cape Canaveral. 10 minutes after lift-off, it reaches its farthest distance north of the equator, 4000 kilometers. Half a cycle later it reaches its farthest distance south of the equator on the other side of the earth, also 4000 km. The Discovery completes an orbit every 90 minutes. Let y be the number of kilometers the space shuttle is north of the equator. (Consider distances south of the equator to be negative.) Let t be the number of minutes since lift-off.
- Sketch a graph of this situation.
 - Write an equation expressing y in terms of t .
 - What is the space ship's distance from the equator when $t = 163$. Is the space shuttle north or south of the equator at this time?
 - Calculate the distance of Cape Canaveral from the equator by finding y when $t = 0$.
 - What is first time at which Discovery is 1600 km south of the equator?

3. You are on board a submarine, submerged in the Pacific Ocean. At time $t = 0$ you make contact with an enemy destroyer. Immediately you start porpoising (going deeper and shallower). At time $t = 4$ minutes you are at your deepest, 1000 meters **below** the surface. (Let the surface of the water be $y = 0$.) At time $t = 9$ minutes you are at your next shallowest point, 200 meters below the surface. Your depth varies sinusoidally with time.

a. Sketch a graph of your depth with respect to time.

b. Write an equation expressing your depth in terms of time.

c. Your submarine is safe from the enemy destroyer when it is 300 meters or more below the surface. Was your submarine safe at time $t = 0$? Justify your answer.

d. Between what two times is your submarine first safe?

4. As you stop your car at a traffic light, a pebble becomes wedged between the tire treads. When you start off, the distance of the pebble from the pavement varies sinusoidally with the distance you have traveled. The period is the circumference of the wheel, and the diameter of the wheel is 24 inches.
- Sketch a graph of this function.

b. Write an equation of this function.

c. Predict the pebble's distance from the pavement when you have driven 35 inches.

d. What are the first two distances you have traveled when the pebble is 11 inches from the pavement?

Name: _____

2.6 Tan, Cot, Sec, Csc Parent Functions

Draw two cycles of each parent function. Fill in the information below each graph. Try to do so without your notes!!

1. tangent

Domain: _____ Range: _____ Period: _____ Line of symmetry: _____

Increasing: _____ Decreasing: _____

2. cotangent

Domain: _____ Range: _____ Period: _____ Line of symmetry: _____

Increasing: _____ Decreasing: _____

3. cosecant

Domain: _____ Range: _____ Period: _____ Line of symmetry: _____

Increasing: _____ Decreasing: _____

4. secant

Domain: _____ Range: _____ Period: _____ Line of symmetry: _____

Increasing: _____ Decreasing: _____