

5.1 Reciprocal, Quotient and Pythagorean Identities

Name: _____

Transform the expression on the left to the expression on the right. Use a separate sheet of paper.

1. $\cos x \tan x$ to $\sin x$

2. $\csc x \tan x$ to $\sec x$

3. $\sec x \cot x \sin x$ to 1

4. $\csc x \tan x \cos x$ to 1

5. $\sin^2 x \sec x \csc x$ to $\tan x$

6. $\cos^2 x \csc x \sec x$ to $\cot x$

7. $\tan x + \cot x$ to $\csc x \sec x$

8. $\sin x + \cot x \cos x$ to $\csc x$

9. $\csc x - \sin x$ to $\cot x \cos x$

10. $\sec x - \cos x$ to $\sin x \tan x$

11. $\tan x (\sin x + \cot x \cos x)$ to $\sec x$

12. $\cos x (\sec x + \cos x \csc^2 x)$ to $\csc^2 x$

13. $(1 + \sin x)(1 - \sin x)$ to $\cos^2 x$

14. $(\sec x - 1)(\sec x + 1)$ to $\tan^2 x$

15. $(\cos x - \sin x)^2$ to $1 - 2 \cos x \sin x$

16. $(1 - \tan x)^2$ to $\sec^2 x - 2 \tan x$

17. $(\tan x + \cot x)^2$ to $\sec^2 x + \csc^2 x$

18. $(\cos x - \sec x)^2$ to $\tan^2 x - \sin^2 x$

19. $\frac{\csc^2 x - 1}{\cos x}$ to $\cot x \csc x$

20. $\frac{1 - \cos^2 x}{\tan x}$ to $\sin x \cos x$

21. $\frac{\sec^2 x - 1}{\sin x}$ to $\tan x \sec x$

22. $\frac{1 + \cot^2 x}{\sec^2 x}$ to $\cot^2 x$

23. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}$ to $\cot x$

24. $\frac{\csc x}{\cos x} - \frac{\cos x}{\sin x}$ to $\tan x$

25. $\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x}$ to $2 \csc^2 x$

26. $\frac{1}{\sec x - \tan x} + \frac{1}{\sec x + \tan x}$ to $2 \sec x$

**Monday – ODDS
Tuesday - EVENS**

5.2 Proving Trig Identities

Name: _____

Prove each identity. You may start on either side, but once you start you must work only on one side! Use a separate sheet of paper.

$$1. \sec x (\sec x - \cos x) = \tan^2 x$$

$$2. \tan x (\cot x + \tan x) = \sec^2 x$$

$$3. \sin x (\csc x - \sin x) = \cos^2 x$$

$$4. \cos x (\sec x - \cos x) = \sin^2 x$$

$$5. \csc^2 x - \cos^2 x \csc^2 x = 1$$

$$6. \cos^2 x + \tan^2 x \cos^2 x = 1$$

$$7. (\sec x + 1)(\sec x - 1) = \tan^2 x$$

$$8. (1 + \sin x)(1 - \sin x) = \cos^2 x$$

$$9. \sec^2 x + \tan^2 x \sec^2 x = \sec^4 x$$

$$10. \cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$$

$$11. \cos^4 x - \sin^4 x = 1 - 2\sin^2 x$$

$$12. \sec^4 x - \tan^4 x = 1 + 2\tan^2 x$$

$$13. \frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} = \tan x$$

$$14. \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$$

$$15. \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$$

$$16. \frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$$

$$17. \frac{1}{1 + \cos x} = \csc^2 x - \csc x \cot x$$

$$18. \frac{1}{1 - \sin x} = \sec^2 x + \sec x \tan x$$

$$19. \frac{\cos x}{\sec x - 1} - \frac{\cos x}{\tan^2 x} = \cot^2 x$$

$$20. \frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x} = 2 \csc x$$

$$21. \frac{\sec x}{\sec x - \tan x} = \sec^2 x + \sec x \tan x$$

$$22. \frac{1 + \sin x}{1 - \sin x} = 2 \sec^2 x + 2 \sec x \tan x - 1$$

$$23. \sin^3 x \cos^2 x = \sin^3 x - \sin^5 x$$

$$24. \sin^3 x \cos^2 x = \cos^2 x \sin x - \cos^4 x \sin x$$

$$25. \sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$$

$$26. \sec x + \tan x = \frac{1}{\sec x - \tan x}$$

$$27. \frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x} = \frac{1 - 4\cos x}{1 - \cos x}$$

$$28. \frac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5} = \frac{\tan x - 4}{\tan x + 2}$$

Thursday – Odds
Friday - Evens

4.3 Sum and Difference Properties (1)

Name: _____

Use the cofunction properties to find an equivalent expression:

1. $\sin 52^\circ$

2. $\cot 43^\circ$

3. $\sec \frac{2\pi}{5}$

4. $\cos \frac{\pi}{4}$

5. $\csc \frac{3\pi}{8}$

Rewrite each expression with a positive argument using the odd/even properties:

6. $\sin(-70^\circ)$

7. $\sec(-25^\circ)$

8. $\tan\left(-\frac{5\pi}{7}\right)$

9. $\cos\left(-\frac{\pi}{5}\right)$

10. $\cot(-35^\circ)$

11. $\csc\left(-\frac{8\pi}{13}\right)$

Simplify the given expression using the sum and difference properties:

12. $\sin 5 \cdot \cos 3 - \cos 5 \cdot \sin 3$

13. $\cos 37^\circ \cdot \cos 23^\circ - \sin 37^\circ \cdot \sin 23^\circ$

14.
$$\frac{\tan 2x + \tan 5x}{1 - \tan 2x \cdot \tan 5x}$$

15. $\cos(x+y) \cdot \cos y + \sin(x+y) \cdot \sin y$

16. $\sin \frac{5\pi}{12} \cos \frac{\pi}{12} - \cos \frac{5\pi}{12} \sin \frac{\pi}{12}$

17.
$$\frac{1 + \tan 3x \tan 4x}{\tan 3x - \tan 4x}$$

4.4 Sum and Difference Properties (2)

Name: _____

Angles A and B are in standard position. Let $\sin A = \frac{2}{3}$, $\cos A > 0$ and $\tan B = \frac{4}{3}$, $\cos B < 0$.

Draw angles A and B in the appropriate quadrants and find the following:

1. $\cos(A - B)$

2. $\sin(A - B)$

3. $\tan(A - B)$

Angles A and B are in standard position. Let $\sin A = \frac{1}{3}$, $\cos A < 0$ and $\tan B = -\frac{2}{3}$, $\cos B > 0$.

Draw angles A and B in the appropriate quadrants and find the following:

4. $\sin(A + B)$

5. $\cos(A + B)$

6. $\tan(A + B)$

Find the exact value. Don't make it harder than it needs to be!

7. $\cos 15^\circ$

8. $\tan 15^\circ$

9. $\cot 15^\circ$

10. $\sec 15^\circ$

11. $\sin 75^\circ$

12. $\cot 75^\circ$

13. $\tan 75^\circ$

14. $\csc 75^\circ$

15. $\sec 75^\circ$

4.5 Sum and Difference Properties (3)

Name: _____

Show that the trig functions do NOT distribute over addition and subtraction by letting $A = 60^\circ$ and $B = 90^\circ$:

1. $\sin(A+B) \neq \sin A + \sin B$

2. $\cot(A-B) \neq \cot A - \cot B$

3. $\sec(A+B) \neq \sec A + \sec B$

4. $\csc(A-B) \neq \csc A - \csc B$

Demonstrate that the given property is true by substituting $A = \frac{2\pi}{3}$ and $B = \frac{\pi}{6}$:

5. $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

6. $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

Prove that the given equation is an identity:

$$7. \cos(\theta - 90^\circ) = \sin \theta$$

$$8. \sec\left(x - \frac{\pi}{2}\right) = \csc x$$

$$9. \tan(x - 90^\circ) = -\cot x$$

$$10. \cos\left(x - \frac{3\pi}{2}\right) = -\sin x$$

$$11. \sin(x + \pi) = -\sin x$$

$$12. \frac{\cos(x - y)}{\sin x \cos y} = \cot x + \tan y$$

$$13. \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{3}\right) = \cos x$$

$$14. \sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$15. (\cos A \cos B - \sin A \sin B)^2 + (\sin A \cos B + \cos A \sin B)^2 = 1$$

Hint: Don't foil!

PreAP PreCalculus
4.6 Proofs Jigsaw

Name: _____
My number (1-5): _____

You will be assigned a number #1-5. You are to become the “expert” on this question by working with others who are assigned your number. Then, you will report back to your original group and teach them how to do your question. They will then teach you how to do each question! Any question not covered by your group needs to be completed by the end of class!

$$1. \tan^2 x + 1 + \tan x \sec x = \frac{1 + \sin x}{\cos^2 x}$$

$$4. \frac{\sin(x - y)}{\sin(x + y)} = \frac{\tan x - \tan y}{\tan x + \tan y}$$

$$2. \tan x + \cot x = \sec x \csc x$$

$$5. \sin(x + y) + \sin(x - y) = 2 \sin x \cos y$$

$$3. \frac{\sin^2 x}{\cos^2 x + 3 \cos x + 2} = \frac{1 - \cos x}{2 + \cos x}$$

$$6. \sin(\pi + x) = -\sin x$$