**Name:____________** **4.1 Converting & Evaluating Logs**

# We did a quizlet live in class over this material! Do questions #1, 3, 4, 5, 6, 7, and 8 on the online quiz!

A. Express in log form.

1. $2^3 = 8$
2. $10^{-2} = \frac{1}{100}$
3. $e^0 = 1$
4. $2^{10} = 1024$
5. $4^{\frac{1}{2}} = 2$
6. $27^{\frac{1}{3}} = 3$
7. $625^{\frac{3}{4}} = 125$
8. $4^{-\frac{3}{2}} = \frac{1}{8}$

B. Express in exponential form.

9. $\log_2 16 = 4$
10. $\ln\ 1 = 0$
11. $\log_{\frac{1}{3}} 27 = -3$
12. $\log_{\frac{1}{2}} \frac{1}{4} = 2$
13. $\log \frac{1}{10} = -1$
14. $\log 1000 = 3$
15. $\log_2 \frac{1}{8} = -3$
16. $\log_2 64 = 6$

C. Evaluate.

17. $\log 10,000$
18. $\log_2 32$
19. $\log_3 27$
20. $\log 10$
21. $\log \frac{1}{100,000}$
22. $\log_2 \frac{1}{4}$
23. $\log_3 1$
24. $\log_3 \frac{1}{9}$
25. $\log_{\frac{1}{3}} 27$
26. $\log_{\frac{1}{2}} 8$
27. $\log_{27} 81$
28. $\log_8 32$
29. $\log_{\sqrt{2}} 16$
30. $\log \sqrt{10}$
31. $\log_{\frac{1}{3}} 81$
32. $\log_{\frac{1}{2}} \frac{1}{16}$

Name:___________________

# 4.2 Laws of Logs/Change of Base

We did a quizizz in the hallway over this material!!

Use laws of logs to expand the expression

1. $\log_2(2x)$

2. $\log_2(x(x-1))$

3. $\log_2(AB^2)$

4. $\log_3(x\sqrt{y})$

5. $\log_5\sqrt[3]{x^2+1}$

6. $\log\left(\dfrac{x^3y^4}{z^6}\right)$

7. $\log_2\left(\dfrac{x(x^2+1)}{\sqrt{x^2-1}}\right)$

8. $\ln\left(x\sqrt{\dfrac{y}{z}}\right)$

9. $\log\sqrt[4]{x^2+y^2}$

10. $\ln\left(\dfrac{x^3\sqrt{x-1}}{3x+4}\right)$

Use the laws of logarithms to condense the expression.

11. $\log_3 5 + 5\log_3 2$

12. $\log 12 + \frac{1}{2}\log 7 - \log 2$

13. $\log_2 A + \log_2 B - 2\log_2 C$

14. $\log_5\left(x^2 - 1\right) - \log_5(x - 1)$

15. $4\log x - \frac{1}{3}\log\left(x^2 + 1\right) + 2\log(x - 1)$

16. $\ln(a + b) + \ln(a - b) - 2\ln c$

17. $\ln 5 + 2\ln x + 3\ln\left(x^2 + 5\right)$

18. $2\left(\log_5 x + 2\log_5 y - 3\log_5 z\right)$

19. $\frac{1}{3}\log(2x + 1) + \frac{1}{2}\left[\log(x - 4) - \log\left(x^4 - x^2 - 1\right)\right]$

20. $\log_a b + c\log_a d - r\log_a s$

Use the change of base formula and a calculator to evaluate the logarithm, correct to six decimal places.

21. $\log_2 5$

22. $\log_5 2$

23. $\log_3 16$

24. $\log_6 92$

# 4.3 Logarithmic Equations

Name:____________________

Solve for x. Do not use a calculator.

1. $2^{5x+2} = 2^{3x-4}$

2. $4^{x-5} = \frac{1}{4}$

3. $3^{x-1} = \sqrt{3}^{x+1}$

4. $(\frac{1}{8})^{x-1} = (\frac{1}{4})^{1-x}$

5. $\log_x 81 = 2$

6. $\log_3 x = -3$

7. $\log_4 x = \frac{-5}{2}$

8. $\log_8 x = \frac{-4}{3}$

9. $\log_x 81 = -2$

10. $\log_x 64 = -3$

11. $\log_{\sqrt{2}} x = 8$

12. $\log_2 (3x - 4) = 3$

13. $\log_5 x = 2\log_5 10$

14. $\ln x = \ln 2 - \ln 5$

15. $\ln x = \ln e^2 - 1$

16. $\ln(x - 2) - \ln 2 = \ln 3 - \ln(x - 1)$

17. $e^x = 1$

18. $e^x = 2$

19. $\ln x + \ln(5 - x) = \ln 2 + \ln 3$

20. $\ln x = \sqrt{3}$

21. $\log x + \log(x - 9) = 1$

22. $\log_3(x - 4) + \log_3(x + 4) = 2$

23. $2^x = 10$

24. $2^x = 3^{x-1}$

25. $3^{x+2} = 5^{x-1}$

4.4 Logarithmic Equations With a Calculator  Name ____________

**Solve each equation for x. You may use a calculator on #4-21, but you must show all work before using a calculator. Round your answer to the nearest thousandth.**

1. $\log_2(\log_2(\log_2 16)) = x$

2. $\log(\log_2(\log_3 9)) = x$

3. $\log_4(\log_2(\log_2 16)) = x$

4. $3^x = 12$

5. $4^x = 7$

6. $21^x = 7$

7. $e^{2x} = 4$

8. $e^{\frac{1}{2}x} = 6$

9. $e^x = 2$

10. $\ln x = 7$

11. $\ln x = \sqrt{3}$

12. $\ln x = -5$

13. $3e^{5x} + 2 = 7$

14. $3e^{3x-2} + 4 = 12$

15. $2e^{\frac{x}{3}} - 2 = 4$

16. $2^{x} = 7^{x-2}$

17. $3^{x} = 4^{x-1}$

18. $5^{x-1} = 7^{x+2}$

19. $6\ln(4x) - 1 = 14$

20. $2\ln(x-2) + 3 = 10$

21. $\dfrac{\ln\frac{x}{3}}{4} + 2 = 3$

Name:____________________

# 4.5 Exponential and Log Graphs

**19–24** ■ Match the exponential function with one of the graphs labeled I–VI.

**19.** $f(x) = 5^x$

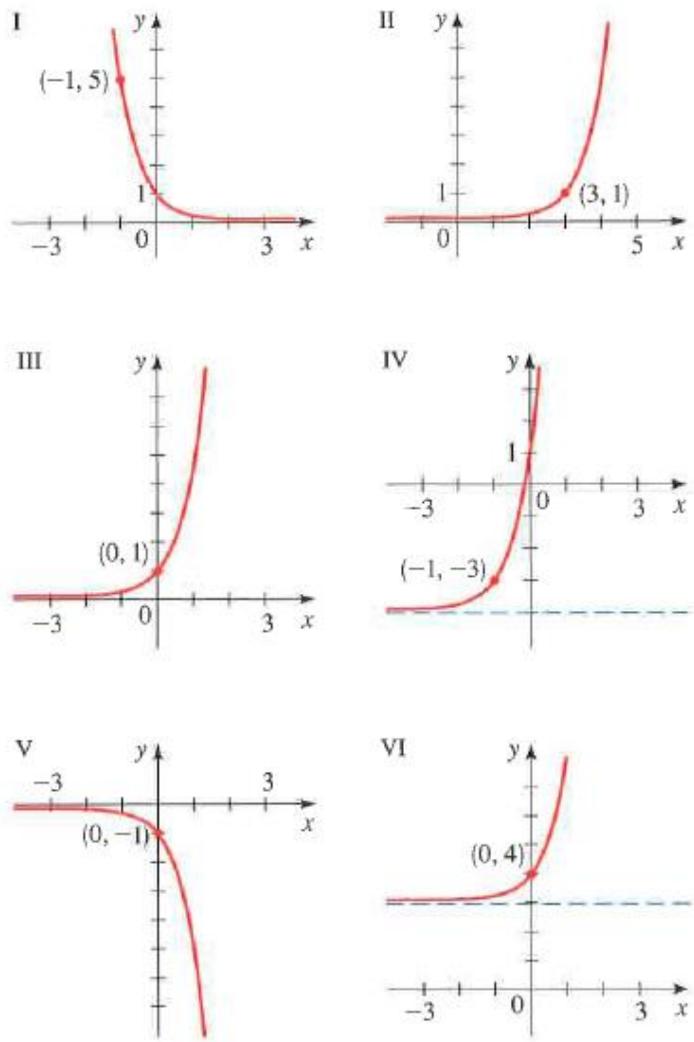
**20.** $f(x) = -5^x$

**21.** $f(x) = 5^{-x}$

**22.** $f(x) = 5^x + 3$

**23.** $f(x) = 5^{x-3}$

**24.** $f(x) = 5^{x+1} - 4$




**25–38** ■ Graph the function, not by plotting points, but by starting from the graphs in Figures 2 and 5. State the domain, range, and asymptote.

**25.** $f(x) = -3^x$

**27.** $g(x) = 2^x - 3$

**29.** $h(x) = 4 + \left(\frac{1}{2}\right)^x$

**31.** $f(x) = 10^{x+3}$

**33.** $f(x) = -e^x$

**35.** $y = e^{-x} - 1$

**37.** $f(x) = e^{x-2}$

**41–46** ■ Match the logarithmic function with one of the graphs labeled I–VI.

**41.** $f(x) = -\ln x$

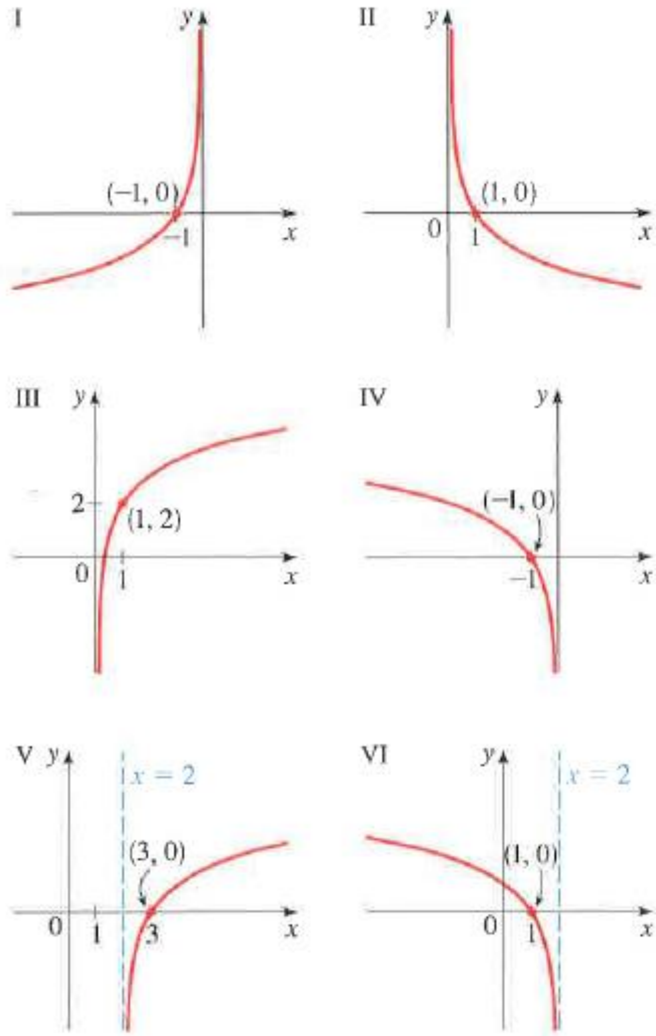
**42.** $f(x) = \ln(x - 2)$

**43.** $f(x) = 2 + \ln x$

**44.** $f(x) = \ln(-x)$

**45.** $f(x) = \ln(2 - x)$

**46.** $f(x) = -\ln(-x)$




**59–64** ■ Find the domain of the function.

**59.** $f(x) = \log_{10}(x + 3)$

**60.** $f(x) = \log_5(8 - 2x)$

**61.** $g(x) = \log_3(x^2 - 1)$

**62.** $g(x) = \ln(x - x^2)$

**Name:________________________**

## 4.6 Logarithmic Applications – Growth and Decay

Show all work on a separate sheet of paper!!!

$$y_t = y_0 e^{rt}$$

$y_t$ – amount after time t, $y_0$ starting amount, r- rate: if you don't know the rate, you need to solve for it first!!!

**Science Applications (Group 1)**

1. The number of bacteria increases from 5,000 to 15,000 in 10 hours.
   a) What is the bacteria's growth rate?
   b) How many bacteria will there be in 20 hours?
   c) When will there be 50,000 bacteria?
2. The number of bacteria in a certain culture increases from 600 to 1800 in 2 hours. How many bacteria will there be after 4 hours?
3. A bacteria culture has an initial count estimate of 4000. After 20 minutes the count is 22, 400. What is the growth rate, and approximately how many minutes did it take for the culture to double?
4. In 1950 the population of a city was 80,000, and in 1960 it was 100,000.
   a) What is its yearly growth rate?
   b) What is the population in 1980?
   c) What is the population in 2010?
5. The population of a city increases 5% per year. If the present population is 500,000, what will the population be in 10 years?

**Population Models (Group 2)**

1. The population of a certain city was 112,000 in 1998, and the observed relative growth rate is 4% per year.
   a. Find a function that models the population after t years.
   b. Find the projected population in the year 2004.
   c. In what year will the population reach 200,000?
2. The frog population in a small pond grows exponentially. The current population is 85 frogs, and the relative growth rate is 18% per year.
   a. Find a function that models the population after t years.
   b. Find the projected population after 3 years.
   c. Find the number of years required for the frog population to reach 600.
3. A culture contains 1500 bacteria and doubles every 30 minutes.
   a. Find a function that models the number of bacteria n(t) after t minutes.
   b. Find the number of bacteria after 2 hours.
   c. After how many minutes will the culture contain 4000 bacteria?
4. A culture starts with 8600 bacteria. After one hour the count is 10,000.
   a. Find a function that models the number of bacteria n(t) after t hours.
   b. Find the number of bacteria after 2 hours.
   c. After how many hours will the number of bacteria double?
5. The population of the world was 5.7 billion in 1995 and the observed relative growth rate was 2% per year.
   a. By what year will the population have doubled?
   b. By what year will the population have tripled?
6. The population of California was 10,586,223 in 1950 and 23,668,562 in 1980. Assume the population grows exponentially.
   a. Find a function that models the population t years after 1950.
   b. Find the time required for the population to double.
   c. Use the function from part a to predict the population of California in the year 2000. Look up California's actual population in 2000, and compare.

**Half-Life (Group 3)**

1. A radioactive substance has a half-life of 420 years. How much remains of a 2 oz. sample after 200 years?

2. An isotope of sodium has a half-life of 15 hours. How many hours will it take for 40% of a given amount to remain?

3. Radium has a half-life of 1600 years. If you begin with 50 mg, when will there be only 20 mg left?

4. The polonium isotope $^{210}Po$ has a half-life of 140 days. If the initial sample is 20 mg, how much will be left after two weeks?

5. If ¼ of a radioactive substance disintegrates in 10 days, what is its half-life?

6. A physicist has 12 grams of radioactive bismuth, which has a half-life of 5 days.
   a) How much will be left after 15 days?
   b) How long will it take for 2 grams to disappear?

**Mo' Money, Mo' Problems (Group 4)**

$A = P(1+\frac{r}{n})^{nt}$ Interest Compounded n times a year $A = Pe^{rt}$ Continuously Compounded Interest

1. $600 is deposited in an account that pays 7% annual interest compounded quarterly.
   a)What is the balance after 5 years?

   b) What would the balance be if it had been compounded continuously?

2. $3000 is invested in an account that pays 5% annual interest. How much more money would you make if the interest was compounded continuously for 10 years instead of compounded quarterly?

3. How many years will it take an investment of $1000 to triple itself when interest is compounded continuously at 5% annually?

4. Money left in a savings account grows exponentially with time. Suppose that you invest $1000 and find that a year later you have $1100. (interest is compounding continuously)
   a) How much will you have after 2 years? 3 years? 4 years?
   b) In how many years will your investment double?

5. $100 is deposited where interest is compounded continuously. Find the balance of the account after 18 months if the interest rate is 5% annually.

6. If you are willing to pay back $5000 within 1 year, how much could you borrow at 13% annual interest if the interest will be compounded
   a) continuously?
   b) monthly?

Name:____________________ Log and Exponent Review

**Laws of Logs:**

Rewrite the exponential equation as a log

**1.** $10^x = y$ **2.** $3^4 = 81$ **3.** $e^7 = x$ **4.** $e^x = 12$

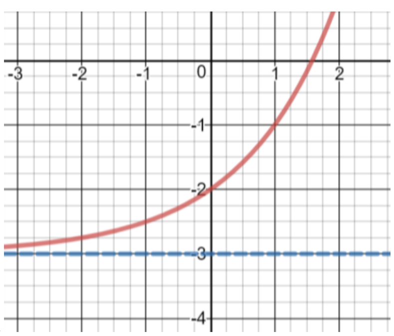
Use the laws of logarithms to expand the expression

**5.** $\log_2 (xy)^{10}$ **6.** $\log_a \left(\dfrac{x^2}{yz^3}\right)$ **7.** $\ln \sqrt[3]{3r^2 s}$ **8.** $\log_5 \sqrt{\dfrac{x-1}{x+1}}$

Use the laws of logarithms to combine the expression into a single log

**9.** $\log_5 \left(x^2 - 1\right) - \log_5 (x-1)$ **10.** $\ln(a+b) + \ln(a-b) - 2\ln c$ **11.** $2(\log_5 x + 2\log_5 y - 3\log_5 z)$

**Exponent and Log Graphs:** Match the graph with the equation



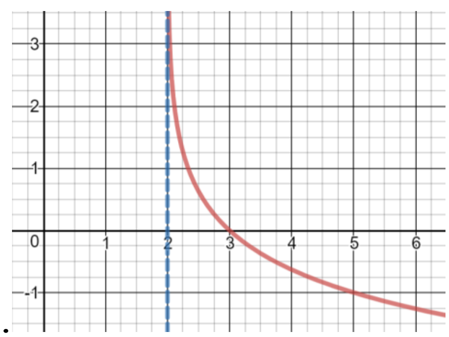

**1.**

A. $f(x) = 2^x - 3$

B. $f(x) = 2^{x-3}$

C. $f(x) = -2^x - 3$

D. $f(x) = 2^{-x} - 3$



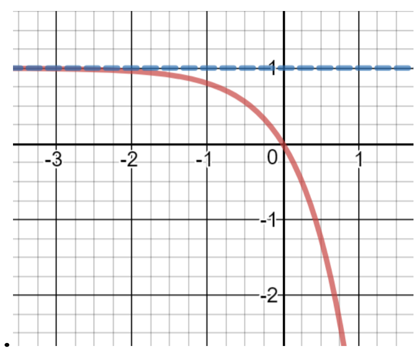

**2.**

A. $f(x) = \log_3(x+2)$

B. $f(x) = \log_3(x-2)$

C. $f(x) = -\log_3(x-2)$

D. $f(x) = -\log_3(x+2)$



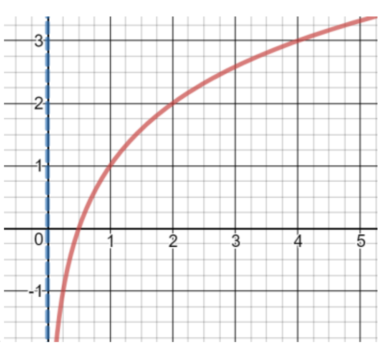

**3.**

A. $f(x) = 5^x + 1$

B. $f(x) = -5^x + 1$

C. $f(x) = \log_5 x + 1$

D. $f(x) = -\log_5 x + 1$



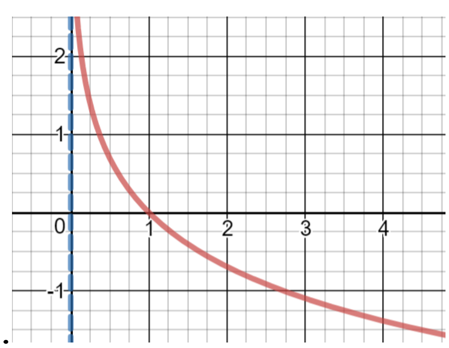

**4.**

A. $f(x) = -\log_2 x + 1$

B. $f(x) = \log_2 x + 1$

C. $f(x) = -2^x + 1$

D. $f(x) = 2^{x+1}$




**5.**

A. $f(x) = -\ln x$

B. $f(x) = \ln(-x)$

C. $f(x) = e^{-x}$

D. $f(x) = e^x$


**6.**

A. $f(x) = 3^x + 1$

B. $f(x) = -3^x + 1$

C. $f(x) = -3^{x+1}$

D. $f(x) = 3^{-x} + 1$

Find the domain of the function in interval notation

7. $f(x) = \log_5(8 - 2x)$

8 $f(x) = \ln(3x + 1)$

**Word Problems**: This is the calculator part. Work the word problems from last week that you haven't done already

**Solving Log Equations/Exponent Rules**:

1. $\left(\frac{16}{81}\right)^{-3/4}$

2. $4^{x+2} = 16^{2x+7}$

3. $\log_4 x = -\frac{1}{2}$

4. $\log_x 8 = -3$

5. $\log_x 64 = 3$

6. $\log_2 x = -2$

7. $\log_{16} x = \frac{3}{4}$

8. $\log_x 3 = \frac{1}{3}$

9. $\log_{400} 1$

10. $\log 1{,}000$

11. $\ln e^4$

12. $\left(\sqrt{2}\right)^{10}$

13. $\log_{16} 32$

14. $\log_2(3x - 4) = 3$

15. $\log\left(x^2 + 3x\right) = 1$

16. $\log x = \frac{1}{2}\log 81 - \frac{1}{3}\log 27$

17. $\log_5 2x = 3\log_5 2 - \log_5 12$

18. $\log_3 x + \log_3(x - 8) = 2$

**Solve each equation. Leave your answer in terms of log, ln or e**

19. $6^x = 7$

20. $3^{2x-1} = 5$

21. $2^{2x-1} = 7^{x+3}$

22. $4e^{2x-3} - 2 = 14$

23. $4\ln(x-3) + 2 = 22$