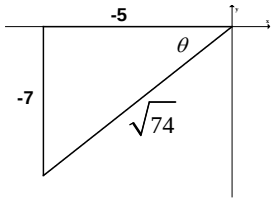
Pre-AP PreCalculus Fall Semester Exam Review ***SOLUTIONS***

1. 

$\sin\theta = -\dfrac{7\sqrt{74}}{74}$ $\csc\theta = -\dfrac{\sqrt{74}}{7}$

$\cos\theta = -\dfrac{5\sqrt{74}}{74}$ $\sec\theta = -\dfrac{\sqrt{74}}{5}$

$\tan\theta = \dfrac{7}{5}$ $\cot\theta = \dfrac{5}{7}$

2. $\cos 45^\circ \sin 210^\circ - \sin 30^\circ \cos 135^\circ$

$$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left[-\frac{\sqrt{2}}{2}\right]$$

$$-\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} = 0$$

3. $\cot^2 330^\circ - \csc^2 330^\circ$

$$\left(-\sqrt{3}\right)^2 - \left(-2\right)^2$$

$$3 - 4 = -1$$

4. $\dfrac{\sin 120^\circ}{\cos 120^\circ} = \tan 120^\circ = -\sqrt{3}$

5. $\sin^2 30^\circ + \cos^2 30^\circ + \tan^2 30^\circ - \sec^2 30^\circ$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2$$

$$\frac{1}{4} + \frac{3}{4} + \frac{1}{3} - \frac{4}{3} = \frac{4}{4} - \frac{3}{3} = 0$$

6. $\tan 58.7^\circ \approx 1.645$

7. $\csc 4.9 = x$

$\sin 4.9 = \dfrac{1}{x}$

$x = \dfrac{1}{\sin 4.9}$

$x \approx 11.71$

8. $\theta = \cos^{-1}\left(0.9125\right)$

$\theta \approx 24^\circ$

9. $\theta = \cot^{-1}\left(0.5234\right)$

$\cot\theta = 0.5234$

$\tan\theta = \dfrac{1}{0.5234}$

$\theta = \tan^{-1}\left(\dfrac{1}{0.5234}\right)$

$\theta \approx 62^\circ$

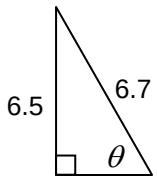
10. $\sec\theta = -12.84$ and $180^\circ < \theta < 270^\circ$

$\cos\theta = -\dfrac{1}{12.84}$

$\theta = \cos^{-1}\left(-\dfrac{1}{12.84}\right)$

$\theta \approx 264^\circ$

11.

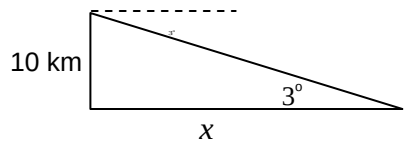


$$\sin\theta = \frac{6.5}{6.7}$$

$$\theta = \sin^{-1}\left(\frac{6.5}{6.7}\right)$$

$$\theta \approx 76^{\circ}$$

12.



a)

$$\tan 3 = \frac{10}{x}$$

$$x = \frac{10}{\tan 3}$$

$$x \approx 190.81 \text{ km}$$

b)

$$\tan\theta = \frac{10}{300}$$

$$\theta = \tan^{-1}\left(\frac{10}{300}\right)$$

$$\theta \approx 2^{\circ}$$

Remember: $y = C + A\sin B(x - D)$ where

$A =$ amplitude

$C =$ vertical displacement (shift)

$D =$ horizontal displacement (shift)

$Period = \frac{360}{B} = \frac{2\pi}{B}$ for sine or cosine

$Period = \frac{180}{B} = \frac{\pi}{B}$ for tangent

Critical Points occur every $\frac{Period}{4}$

13. $y = -10 + 20\sin 2(\theta + 30^{\circ})$

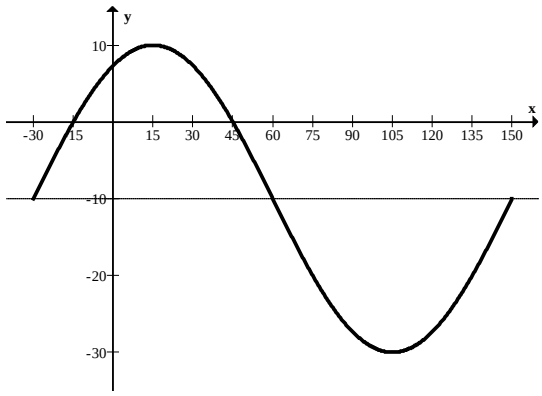
C = -10

A = 20

D = -30 (left 30)

Period = $\frac{360}{2} = 180$

Crit. Pnts = $\frac{180}{4} = 45$



14. $y = 3 + 5\cos\frac{1}{2}(\theta + 90^{\circ})$

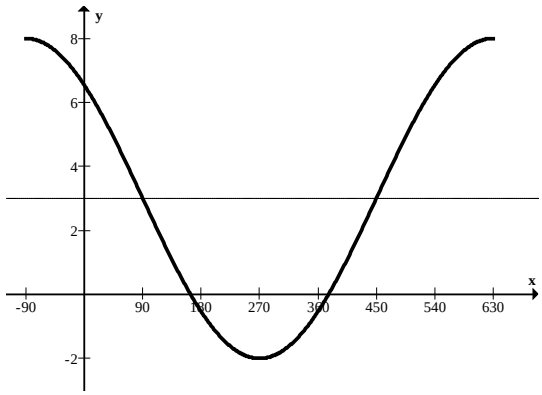
C = 3

A = 5

D = -90 (left 90)

Period = $\frac{360}{1/2} = 720$

Crit. Pnts = $\frac{720}{4} = 180$



15. $y = 3 + 2\cos\frac{1}{5}(x - \pi)$

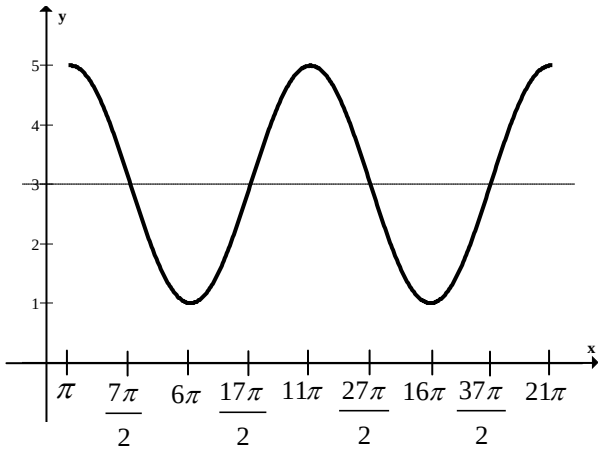
A = 2

C = 3

D = π

Period = $\frac{2\pi}{1/5} = 10\pi$

Crit. Pnts = $\frac{10\pi}{4} = \frac{5\pi}{2}$



16. $y = 2 + 6\sin\frac{\pi}{4}(x - 1)$

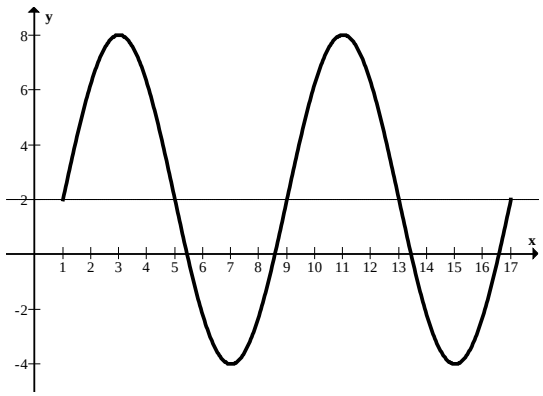
A = 6

C = 2

D = 1

Period = $\frac{2\pi}{\pi/4} = 8$

Crit. Pnts = $\frac{8}{4} = 2$



17. $y = -1 + 3\cot 2(\theta - 30^\circ)$

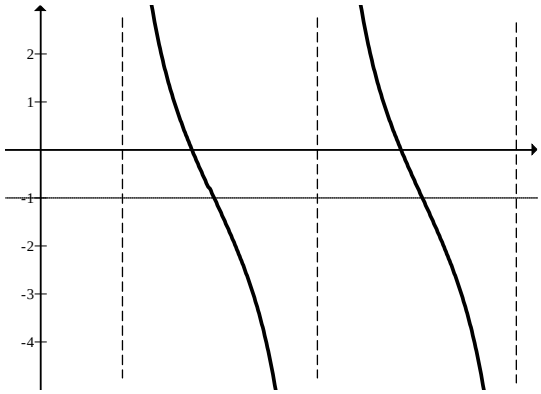
A = 3

C = -1

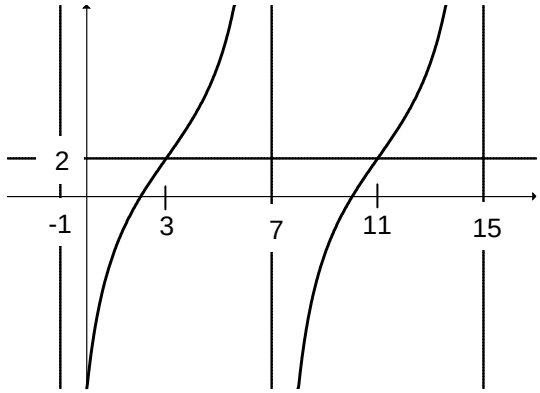
D = 30

Period = $\frac{180}{2} = 90$

Crit. Pnts = $\frac{90}{4} = 22.5$



18. $y = 2 + 5\tan\frac{\pi}{8}(x-3)$



A = 5

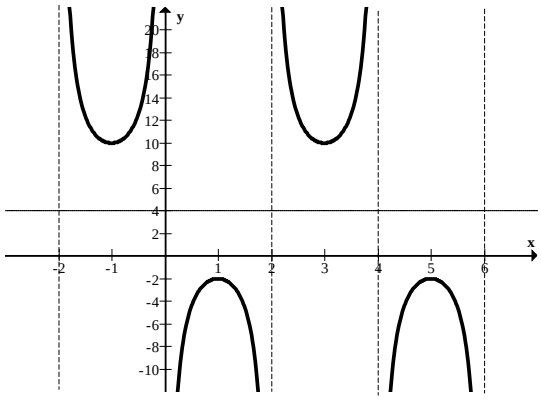
C = 2

D = 3

Period = $\frac{\pi}{\pi/8} = 8$

Crit. Pnts. = $\frac{8}{4} = 2$

19. $y = 4 + 6\sec\frac{\pi}{2}(x+1)$



A = 6

C = 4

D = -1

Period = $\frac{2\pi}{\pi/2} = 4$

Crit. Pnts = $\frac{4}{4} = 1$

20. $y = 1.45 + 1.11\sin 10(x+16)$

$y = 1.45 - 1.11\sin 10(x-2)$

$y = 1.45 + 1.11\cos 10(x+7)$

$y = 1.45 - 1.11\cos 10(x-11)$

To find B $B = \frac{360}{Period} = \frac{360}{36} = 10$

21. $y = -2 + 5\sin\frac{\pi}{15}(x+12.5)$

$y = -2 - 5\sin\frac{\pi}{15}(x-2.5)$

$y = -2 + 5\cos\frac{\pi}{15}(x+5)$

$y = -2 - 5\cos\frac{\pi}{15}(x-10)$

To find B $B = \frac{2\pi}{Period} = \frac{2\pi}{30} = \frac{\pi}{15}$

22. $\sin\frac{\pi}{2}+6\cos\frac{\pi}{3}=(1)+6\left(\frac{1}{2}\right)=1+3=4$

23. $\dfrac{\cos\frac{5\pi}{3}}{\sin\frac{5\pi}{3}}=\cot\frac{5\pi}{3}=-\frac{\sqrt{3}}{3}$

24. $f(x)=5+2\cos\frac{\pi}{4}(x-10)$

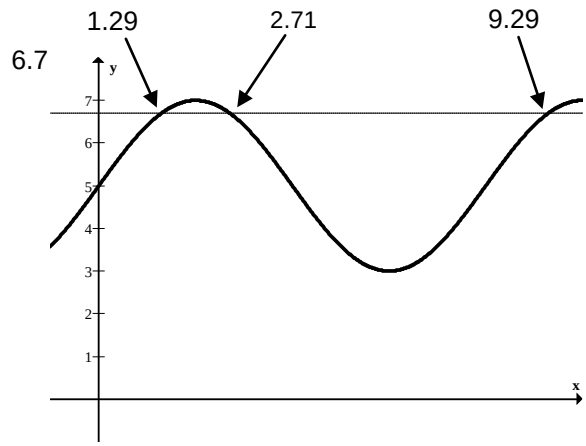
a) $f(17.3)=5+2\cos\frac{\pi}{4}(17.3-10)$

$f(17.3)\approx 6.705$

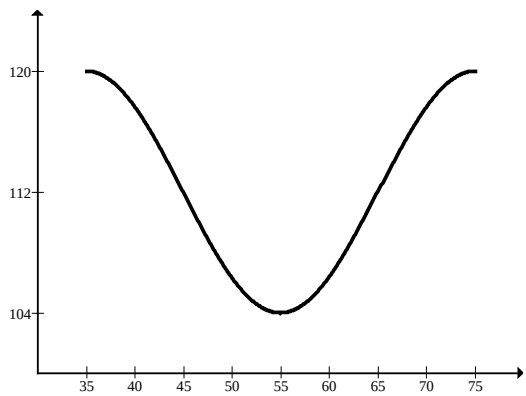
$f(x)=6.7$

b) $6.7=5+2\cos\frac{\pi}{4}(x-10)$

$x=1.29,\quad 2.71,\quad 9.29$



25. a)



b) $y=112+8\cos\frac{\pi}{20}(x-35)$

c) $y(0)\approx 118^{\circ}$

d) $114=112+8\cos\frac{\pi}{20}(x-35)$

$x=3.39,\quad 26.61,\quad 43.39$

26.

$$\frac{1}{1+\cos s}=\csc^2 s-\csc s\cot s$$

$$\left(\frac{1-\cos s}{1-\cos s}\right)\left(\frac{1}{1+\cos s}\right)=$$

$$\frac{1-\cos s}{1-\cos^2 s}=$$

$$\frac{1-\cos s}{\sin^2 s}=$$

$$\frac{1}{\sin^2 s}-\frac{\cos s}{\sin^2 s}=$$

$$\frac{1}{\sin^2 s}-\frac{1}{\sin s}\cdot\frac{\cos s}{\sin s}=$$

$$\csc^2 s-\csc s\cot s=\csc^2 s-\csc s\cot s$$

27.

$$\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}=2\csc x$$

$$\left(\frac{1+\cos x}{1+\cos x}\right)\left(\frac{\sin x}{1-\cos x}\right)+\frac{1-\cos x}{\sin x}=$$

$$\frac{(1+\cos x)\sin x}{1-\cos^2 x}+\frac{1-\cos x}{\sin x}=$$

$$\frac{(1+\cos x)\sin x}{\sin^2 x}+\frac{1-\cos x}{\sin x}=$$

$$\frac{1+\cos x}{\sin x}\cdot\frac{\sin x}{\sin x}+\frac{1-\cos x}{\sin x}=$$

$$\frac{1+\cos x}{\sin x}+\frac{1-\cos x}{\sin x}=$$

$$\frac{2}{\sin x}=$$

$$2\csc x=2\csc x$$

28. $$\frac{1-3\cos x-4\cos^2 x}{\sin^2 x}=\frac{1-4\cos x}{1-\cos x}$$

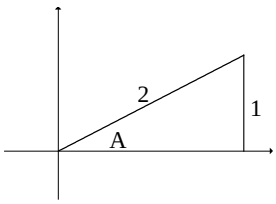
$$\frac{(1-4\cos x)(1+\cos x)}{1-\cos^2 x}=$$

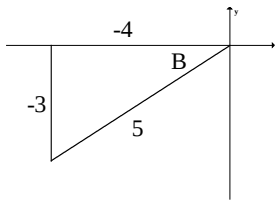
$$\frac{(1-4\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x)}=$$

$$\frac{1-4\cos x}{1-\cos x}=\frac{1-4\cos x}{1-\cos x}$$

29. $\sin A=\frac{1}{2}$ $\qquad$ $\tan B=\frac{3}{4}$

$\cos A>0$ $\qquad$ $\sin B<0$





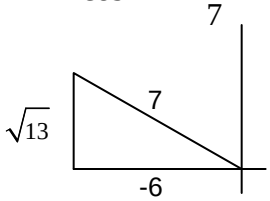
$$\sin(A+B)=\sin A\cos B+\cos A\cos B$$

$$=\left(\frac{1}{2}\right)\left(-\frac{4}{5}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{3}{5}\right)$$

$$=-\frac{4}{10}-\frac{3\sqrt{3}}{10}\ \sqrt{}$$

$$=\frac{-4-3\sqrt{3}}{10}\ \sqrt{}$$

30. $\cos A=-\frac{6}{7}$ $\qquad$ Quadrant II



$$\sin 2A=2\sin A\cos A$$

$$=2\left(\frac{\sqrt{13}}{7}\right)\left(-\frac{6}{7}\right)$$

$$=-\frac{12\sqrt{13}}{49}$$

$$\cos 2A=\cos^2 A-\sin^2 A$$

$$=\left(-\frac{6}{7}\right)^2-\left(\frac{\sqrt{13}}{7}\right)^2$$

$$=\frac{36}{49}-\frac{13}{49}$$

$$=\frac{23}{49}$$

$$\tan 2A=\frac{\sin 2A}{\cos 2A}=\frac{-\frac{12\sqrt{13}}{49}}{\frac{23}{49}}=-\frac{12\sqrt{13}}{23}$$

31. $\cos\theta=-\frac{3}{5},\quad 180^\circ<\theta<270^\circ \quad\Rightarrow\quad 90^\circ<\frac{1}{2}\theta<135^\circ \quad\Rightarrow\quad \frac{1}{2}\theta$ is in Quadrant II

$$\sin\frac{1}{2}\theta=+\sqrt{\frac{1}{2}(1-\cos\theta)}=\sqrt{\frac{1}{2}\left(1-\frac{-3}{5}\right)}=\sqrt{\frac{1}{2}\left(\frac{8}{5}\right)}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

$$\cos\frac{1}{2}\theta=-\sqrt{\frac{1}{2}(1+\cos\theta)}=-\sqrt{\frac{1}{2}\left(1+\frac{-3}{5}\right)}=-\sqrt{\frac{1}{2}\left(\frac{2}{5}\right)}=-\sqrt{\frac{1}{5}}=-\frac{1}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$$

$$\tan\frac{1}{2}\theta=\frac{\sin\frac{1}{2}\theta}{\cos\frac{1}{2}\theta}=\frac{\frac{2\sqrt{5}}{5}}{-\frac{\sqrt{5}}{5}}=-\frac{2\sqrt{5}}{\sqrt{5}}=-2$$

32. $\cos x + \sqrt{3}\sin x$

$A = 1 \quad B = \sqrt{3} \quad C = \sqrt{(1)^2 + \left(\sqrt{3}\right)^2} = 2$

$\cos D = \frac{1}{2} \qquad D = 60^{\circ} \text{ or } \frac{\pi}{3}$

$\sin D = \frac{\sqrt{3}}{2}$

$$\cos x + \sqrt{3}\sin x = 2\cos(x - 60)$$
$$= 2\cos\left(x - \frac{\pi}{3}\right)$$

33.

$$\tan x = \frac{1 - \cos 2x}{\sin 2x}$$
$$= \frac{1 - \left(1 - 2\sin^2 x\right)}{2\sin x \cos x}$$
$$= \frac{1 - 1 + 2\sin^2 x}{2\sin x \cos x}$$
$$= \frac{2\sin^2 x}{2\cos x \sin x}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$

34.

$$\tan x \tan \frac{1}{2}x = \sec x - 1$$
$$\frac{\sin x}{\cos x}\left(\frac{1 - \cos x}{\sin x}\right) =$$
$$\frac{1 - \cos x}{\cos x} =$$
$$\frac{1}{\cos x} - \frac{\cos x}{\cos x} =$$
$$\sec x - 1 =$$

35. $2\sin\theta\cos\theta = \sqrt{2}\cos\theta \qquad \theta \in \{\text{real numbers of degrees}\}$

$$2\sin\theta\cos\theta - \sqrt{2}\cos\theta = 0$$
$$\cos\theta\left(2\sin\theta - \sqrt{2}\right) = 0$$

$\cos\theta = 0$

$\theta = \cos^{-1} 0$

$\theta = \pm 90 + 360n$

$2\sin\theta - \sqrt{2} = 0$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \sin^{-1}\frac{\sqrt{2}}{2}$

$\theta = 45 + 360n$

$\theta = 135 + 360n$

36. $2\cos^2 x - 5\cos x + 2 = 0 \qquad x \in [0, 2\pi)$

$(2\cos x - 1)(\cos x - 2) = 0$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \cos^{-1}\frac{1}{2}$

$x = \pm\frac{\pi}{3} + 2\pi n$

$\cos x - 2 = 0$

$\cos x = 2$

$x = \cos^{-1} 2$

$\varnothing$

$x = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

37. $x \in [0, 2\pi)$

$4\sin x\cos x = \sqrt{3}$

$2\sin x\cos x = \frac{\sqrt{3}}{2}$

$\sin 2x = \frac{\sqrt{3}}{2}$

$2x = \sin^{-1}\frac{\sqrt{3}}{2}$

$2x = \frac{\pi}{3} + 2\pi n$

$x = \frac{\pi}{6} + \pi n$

$2x = \left(\pi - \frac{\pi}{3}\right) + 2\pi n$

$2x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{\pi}{3} + \pi n$

$x = \left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}\right\}$

38. $\sin 2\theta\cos 64^\circ + \cos 2\theta\sin 64^\circ = \frac{\sqrt{3}}{2} \qquad x \in [0, 360)$

$\sin(2\theta + 64) = \frac{\sqrt{3}}{2}$

$2\theta + 64 = \sin^{-1}\frac{\sqrt{3}}{2}$

$2\theta + 64 = 60 + 360n$

$2\theta = -4 + 360n$

$\theta = -2 + 180n$

$2\theta + 64 = (180 - 60) + 360n$

$2\theta + 64 = 120 + 360n$

$2\theta = 56 + 360n$

$\theta = 28 + 180n$

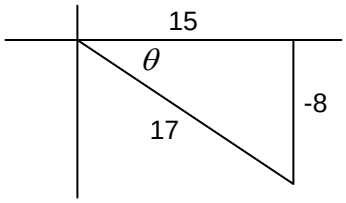
$\theta = \{28^\circ,\ 178^\circ,\ 208^\circ,\ 358^\circ\}$

39. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

40. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

41. $\cos\left(\arcsin\left(-\frac{8}{17}\right)\right)=\frac{15}{17}$

42. $\cos^{-1}\left(\sin\frac{7\pi}{6}\right)=\cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$



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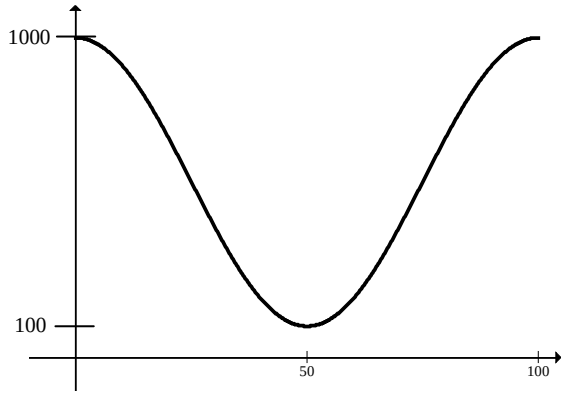


a) $d=550+450\cos\frac{\pi}{50}t$

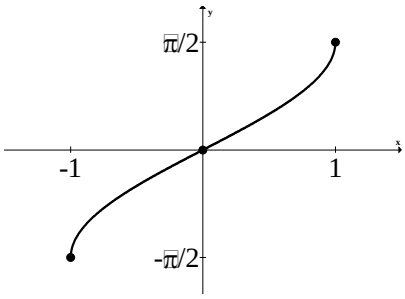
b) $t=\frac{\pi}{50}\cos^{-1}\left(\frac{d-550}{450}\right)$

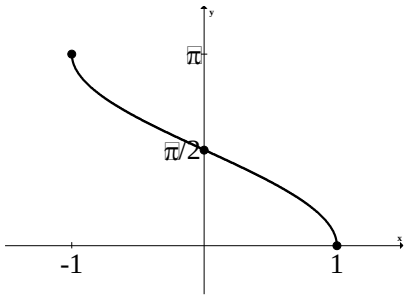
c) $700=550+450\cos\frac{\pi}{50}t$

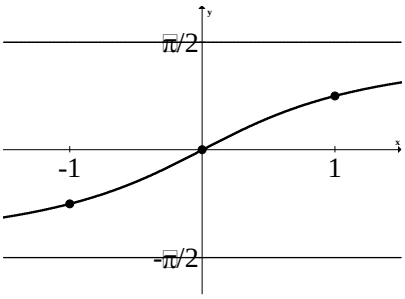
$t=19.59,\ \ 80.41$

$80.41-19.59=60.82$ minutes

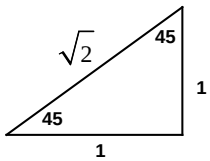
44. a. $y=\sin^{-1}x$ b. $y=\cos^{-1}x$ c. $y=\tan^{-1}x$

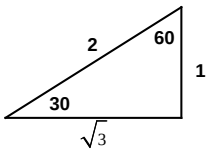


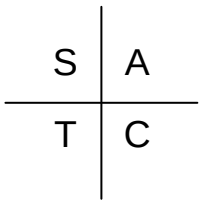




Good To Know







	S	C	T
0	0	1	0
90	1	0	U
180	0	-1	0
270	-1	0	U