13.7 The Intermediate Value Theorem

EQ:

Intermediate Value Theorem

If f is continuous on $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$

Ex. Use the intermediate value theorem to show $f(x)=x^4-3x^2+x-1$ has a zero on $[1,2]$

Ex. Use the intermediate value theorem to find the value of c guaranteed by the theorem.

$f(x)=\dfrac{2x^2+x}{3x-1}$ $\left[\dfrac{3}{2},3\right]$ $f(c)=2$

PreAP PreCalculus

Unit 13

Calculus Part 1

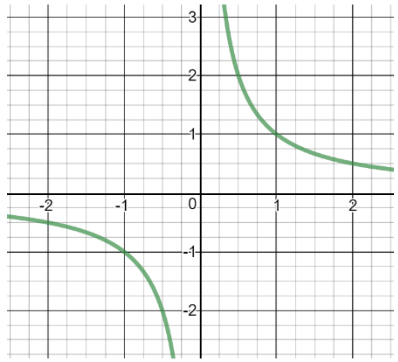
Limits

13.1 Limits

EQ:

$$\lim_{x\to\infty}\frac{1}{x} \qquad \lim_{x\to-\infty}\frac{1}{x}$$

$$\lim_{x\to 1}\frac{1}{x} \qquad \lim_{x\to 0}\frac{1}{x}$$



Definition of a Limit

If $f(x)$ becomes arbitrarily close to a single number L as it approaches c from either side, then the limit of $f(x)$ as x approaches c is L.

$$\lim_{x\to c} f(x) = L$$

Ex. $\lim\limits_{x\to 1} x^2+3$

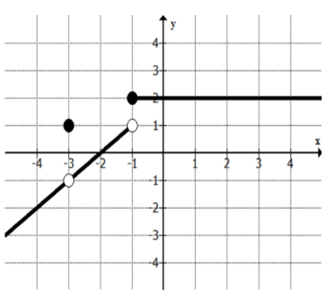
Ex. $\lim\limits_{x\to -1}\dfrac{x+1}{x^2-x-2}$

Ex.

$\lim\limits_{x\to -2} f(x)$

$\lim\limits_{x\to -3} f(x)$

$\lim\limits_{x\to -1} f(x)$



Three cases where the limit DNE

1.

2.

3.

13.6 Special Trig Limits

EQ:

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{x}{\sin x}=1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x}=0 \qquad \lim_{x\to 0}\frac{\cos x-1}{x}=0$$

Ex. $\lim\limits_{y\to 0}\dfrac{\sin 4y}{4y}$

Limits Properties

- $\lim\limits_{x\to c} 4f(x)$ — ex. $\lim\limits_{x\to 3} 7x$
- $\lim\limits_{x\to c} f(x)g(x)$ — ex. $\lim\limits_{x\to 9} x\sqrt{x}$

Ex. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}$

Ex. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x}$

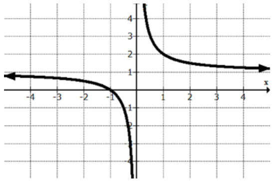
Ex. $\lim\limits_{x\to 0}\dfrac{\sin^2 3x}{\sin^2 7x}$

Ex. $\lim\limits_{h\to 0}\dfrac{\sin 4h}{h^3}$

Ex. $\lim\limits_{x\to 0}\dfrac{1-\cos^2 x}{x\sin 2x}$

13.5 Infinite Limits and Limits at Infinity

EQ:



$\lim_{x \to \infty}$

$\lim_{x \to -\infty}$

Limits at Infinity

1. $\lim_{x \to \infty} \frac{x}{2x-1} =$

2. $\lim_{x \to -\infty} \frac{x}{2x-1} =$

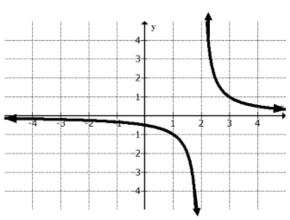
<u>Finding Horizontal Asymptotes</u>

3. $\lim_{x \to -\infty} \frac{5x^2+2}{x-2x^2} =$

4. $\lim_{x \to \infty} \frac{4x}{(x-1)(x+2)} =$

5. Find the horizontal asymptote of $\frac{7x}{\sqrt{x^2+4}}$

Infinite Limits



$\lim_{x \to 2^+}$

$\lim_{x \to 2^-}$

$\lim_{x \to 2}$

6. $\lim_{x \to 0^+} \frac{1}{x}$ $\qquad \lim_{x \to 0^-} \frac{1}{x}$

7. $\lim_{x \to 2^-} \frac{-3}{x-2}$

8. $\lim_{x \to 5^-} \frac{-4}{(x-5)^2}$

13.2 Limits Day 2

EQ:

Ex. $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

Ex. $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$

Ex. $\lim_{x \to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$

Ex. $\lim_{x \to 7} \frac{\sqrt{x+2}-3}{x-7}$

Ex. $f(x) = \begin{cases} 2x-2 & x \neq 1 \\ 8 & x = 1 \end{cases}$

$f(1)$

$\lim_{x \to 0} f(x) =$

$\lim_{x \to 1} f(x) =$

Ex. $f(x) = \begin{cases} 2 & x < -2 \\ 2x+4 & -2 \le x \le 1 \\ 6x & x > 1 \end{cases}$

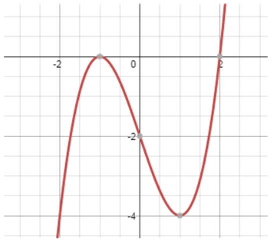
$\lim_{x \to -2} f(x) =$

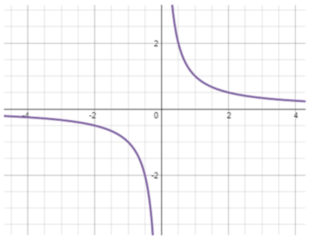
$\lim_{x \to 0} f(x) =$

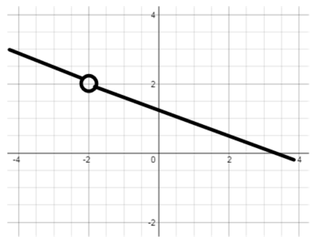
$\lim_{x \to 1} f(x) =$

13.3 Continuity

EQ:



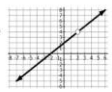




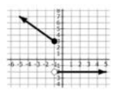
A function is continuous if…

1. $f(c)$ is defined.
2. $\lim_{x\to c} f(x)$ exists.
3. $\lim_{x\to c} f(x) = f(c)$

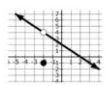
1.



2.



3.



Ex. Is $f(x) = x^3 + 1$ continuous at x=2?

Types of Discontinuity

1. Removable discontinuity (hole)
2. Vertical asymptotes
3. Breaks/jumps

Ex. Is $f(x) = \dfrac{x^2 - x - 6}{x^2 - 2x - 3}$ continuous at x=3?

Ex. Is $f(x) = \begin{cases} x-2 & x > 3 \\ -2x+2 & x \le 3 \end{cases}$ continuous at x=3?

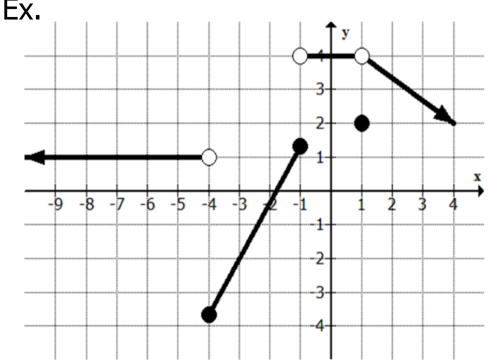
Ex. Is $f(x) = \begin{cases} 2x-1 & x > 5 \\ x^2+16 & x \le 5 \end{cases}$ continuous at x=5?

13.4 One-Sided Limits

EQ:

$\lim_{x\to 0} \dfrac{|x|}{x}$ $\qquad$ $\lim_{x\to 5} \dfrac{|x-5|}{x-5}$ $\qquad$ $\lim_{x\to 4} \dfrac{|x-5|}{x-5}$

Ex.



$\lim_{x\to 0} =$

$\lim_{x\to -1} =$ $\qquad$ $\lim_{x\to -1^+} =$ $\qquad$ $\lim_{x\to -1^-} =$

$\lim_{x\to -4^+} =$ $\qquad$ $\lim_{x\to -4^-} =$ $\qquad$ $\lim_{x\to -4} =$

$\lim_{x\to 1^+} =$ $\qquad$ $\lim_{x\to 1^-} =$ $\qquad$ $\lim_{x\to 1} =$

$f(1) =$ $\qquad$ $f(-1) =$

Ex. $f(x) = \begin{cases} 7 & x < -1 \\ -x+6 & -1 \le x \le 3 \\ 2x & x > 3 \end{cases}$

$\lim_{x\to -1^+} =$ $\qquad$ $\lim_{x\to 3^-} =$

$\lim_{x\to -1^-} =$ $\qquad$ $\lim_{x\to 3^+} =$

$\lim_{x\to -1} =$ $\qquad$ $\lim_{x\to 3} =$

$f(3) =$