Limits Test Review

① $\lim_{x\to 3} \frac{x-3}{x^2-9}$

$\frac{0}{0}$ $\lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(x+3)}$

$\frac{1}{3+3} = \boxed{\frac{1}{6}}$

② $\lim_{x\to 0} \frac{\sin 3x}{\sin 5x} = \lim_{x\to 0} \frac{3x \sin 3x}{3x\ 1} \cdot \lim_{x\to 0} \frac{5x\ 1}{5x \sin 5x}$

$\frac{0}{0}$

$= 3x \lim_{x\to 0} \left|\frac{\sin 3x}{3x}\right| \cdot \frac{1}{5x} \lim_{x\to 0} \left|\frac{5x}{\sin 5x}\right|$

$= 3x(1)\left(\frac{1}{5x}\right)(1) = \frac{3\cancel{x}}{5\cancel{x}} = \boxed{\frac{3}{5}}$

③ $\lim_{h\to 0} \frac{-5(x+h)+5x}{h}$

$\frac{0}{0}$ $\lim_{h\to 0} \frac{\cancel{-5x}-5h+\cancel{5x}}{h}$

$\lim_{h\to 0} \frac{-5\cancel{h}}{\cancel{h}} = \boxed{-5}$

④ $\lim_{x\to 3} \begin{cases} \frac{1}{2}x+1 & x\le 4 \\ 2x-5 & x>4 \end{cases}$

$3<4$ $\quad \frac{1}{2}(3)+1 = \frac{3}{2}+\frac{2}{2} = \boxed{\frac{5}{2}}$

⑤ $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{(\sqrt{x+5}+3)}{(\sqrt{x+5}+3)}$

$\frac{0}{0}$

multiply by conjugate $\lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$

$\lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x+5}+3)}$

$\frac{1}{\sqrt{4+5}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$

⑥ $\lim_{x\to 3^-} \frac{x^2}{x^2-9} = \boxed{-\infty}$

LS 2.9 $\quad \frac{(2.9)^2}{(2.9)^2-9} = \frac{+}{-}$

3 is VA

⑦ $\lim_{x\to 0} \frac{(1-\cos^2 x)}{2x^2} = \lim_{x\to 0} \frac{\sin^2 x}{2x^2}$

$\frac{0}{0}$

$\sin^2 x + \cos^2 x = 1$

$= \frac{1}{2} \cdot \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{\sin x}{x}$

$= \frac{1}{2}(1)(1) = \boxed{\frac{1}{2}}$

⑧ $\lim_{x\to 5^+} \frac{x}{x^2-25} = \boxed{\infty}$

5 is VA

RS 5.1 $\quad \frac{5.1}{(5.1)^2-25} = \frac{+}{+}$

(9) $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$

$\frac{0}{0}$

$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$

$\boxed{2x}$

(10) $\lim_{x\to 2^+} \frac{x-3}{2-x} = \boxed{\infty}$

2 is VA

RS 2.1

$\frac{2.1-3}{2-2.1} = \frac{-}{-} +$

(11) $\lim_{x\to 0} \frac{\sin x \cos x}{x^2-5x}$

$\frac{0}{0}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{\cos x}{x-5}$

$1 \cdot \frac{\cos 0}{0-5} = \boxed{\frac{1}{-5}}$

(12) $\lim_{x\to 3} \frac{\sqrt{x+1}}{x-4}$ Just plug in $x=3$

$\frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \boxed{-2}$

(13) $\lim_{x\to 0} \frac{\frac{1}{x+2}-\frac{1}{2}}{x}$ common denom.

$\frac{0}{0}$

$\lim_{x\to 0} \frac{\frac{2}{2(x+2)}-\frac{(x+2)}{2(x+2)}}{x}$

$\lim_{x\to 0} \frac{\frac{2-(x+2)}{2(x+2)}}{x}$

$\lim_{x\to 0} \frac{\cancel{2}-\cancel{x}-\cancel{2}}{2(x+2)} \cdot \frac{1}{x}$

$\lim_{x\to 0} \frac{-1}{2(x+2)} = \frac{-1}{2(2)} = \boxed{\frac{-1}{4}}$

(14) $\lim_{x\to \infty} \frac{4x-3}{2-x} = \boxed{-4}$

horiz. asymp.

$\frac{\text{same}}{\text{same}}$ $\frac{4}{-1}$

(15) $\lim_{x\to 5} \begin{cases} x-4 & x\le 5 \\ -x+3 & x>5 \end{cases}$

RS

$-5+3=-2$

LS

$5-4=1$

RS ≠ LS

$\boxed{\text{DNE}}$

(16) $\lim_{x\to \infty} \frac{7x^2-2}{(2x-1)(3-x)} = \boxed{-\frac{7}{2}}$

horiz. $\frac{\text{same}}{\text{same}}$

$\frac{7}{-2}$

(17) $\lim_{x\to 2^+} \begin{cases} x^2-2 & x\le 2 \\ 3x-5 & x>2 \end{cases}$

RS >
$3(2)-5 = \boxed{1}$

(18) $\lim_{x\to -\infty} \frac{3x^2-7x}{2x^3} = \boxed{0}$

$\frac{\text{low}}{\text{high}}$ $0=y$

(19) $\lim_{x\to 0} \frac{\sin x(1-\cos x)}{2x^2}$

$\frac{0}{0}$

$\frac{1}{2}\lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1-\cos x}{x}$

$\frac{1}{2}(1)(0) = \boxed{0}$

(20) $\lim_{x\to 1^-} \begin{cases} x^3 & x\le 1 \\ -x+3 & x>1 \end{cases}$

LS <

$(1)^3 = \boxed{1}$

(21) $f(x) = x^3+2x-1$

zero on $[0,1]$

$f(0) = 0^3+2(0)-1 = -1$

$f(1) = 1^3+2(1)-1 = 2$

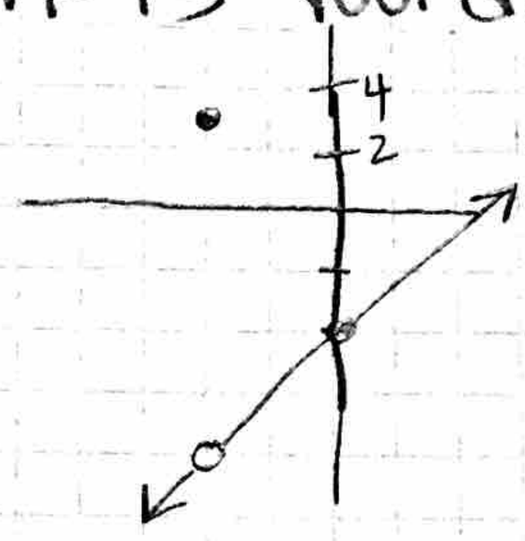
$-1<0<2$ ✓

Remember, your work is your answer for IVT.

(22) $f(x) = \begin{cases} \frac{x^2-4}{x+2} & x\ne -2 \\ 3 & x=-2 \end{cases}$

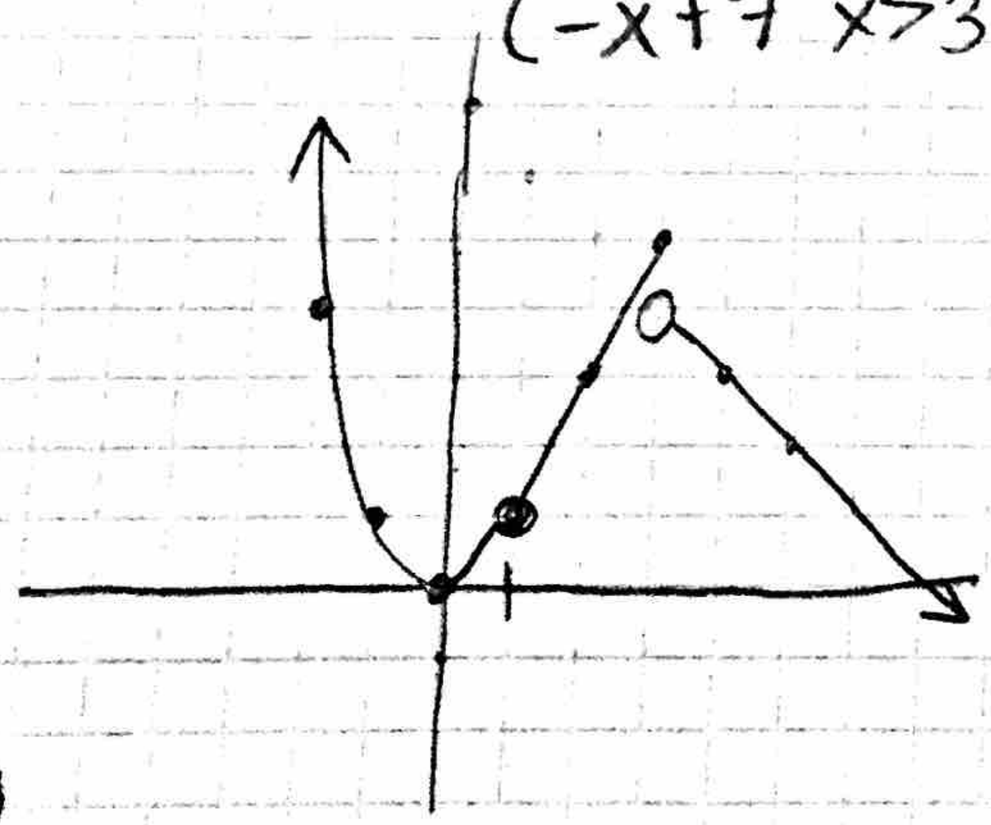
$\frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}$

R.D. @ $x=-2$
$y=-4$



discontinuous at $x=-2$
→ hole

(23) $f(x) = \begin{cases} x^2 & x<1 \\ 2x-1 & 1\le x\le 3 \\ -x+7 & x>3 \end{cases}$



discontinuous @ $x=3$
→ break/jump

(24) $f(x) = \frac{x^2-3x-10}{x^2-25}$

$= \frac{\cancel{(x-5)}(x+2)}{\cancel{(x-5)}(x+5)}$

- hole @ $x=5$
- asymptote at $x=-5$

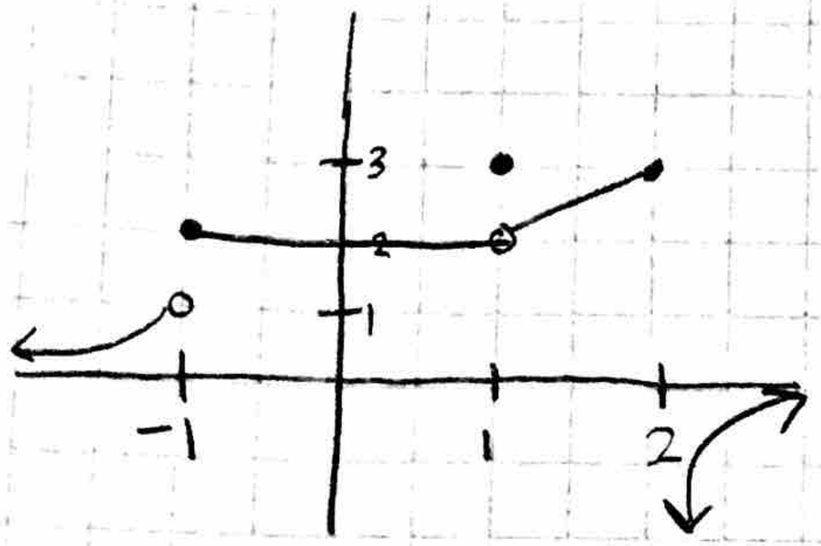
(25) $f(x) = \begin{cases} x^2+1 & x \le 1 \\ cx+4 & x > 1 \end{cases}$

no jumps, each equation should have same y-value at $x=1$

$(1)^2+1 = c(1)+4$

$2 = c+4$

$\boxed{-2 = c}$



(26) $\lim_{x \to -1^+} = \boxed{2}$
RS

(27) $\lim_{x \to 2^+} = \boxed{-\infty}$

(28) $\lim_{x \to -1^-} = \boxed{1}$

(29) $\lim_{x \to 2^-} = \boxed{3}$

(30) $\lim_{x \to 1} = \boxed{2}$
LS 2 RS 2

(31) $\lim_{x \to 0} = \boxed{2}$

(32) $\lim_{x \to -1} = \boxed{\text{DNE}}$
LS 1 RS 2

(33) $g(1) = \boxed{3}$
$x=1$

(34) $\lim_{x \to -\infty} = \boxed{0}$

(35) discont. @

$-1 \to$ break/jump

$1 \to$ removable discontinuity

$2 \to$ vertical asymptote