

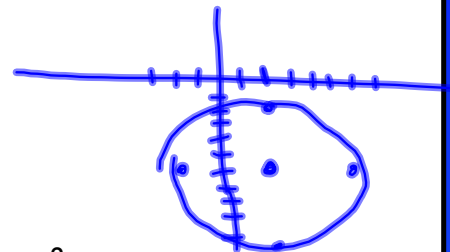
## 11.2 Ellipses

### Warm-Up Monday

Sketch a graph of the circle.

$$\frac{2(x-2)^2}{2} + \frac{2(y+6)^2}{2} = \frac{36}{2}$$
$$(x-2)^2 + (y+6)^2 = 18$$

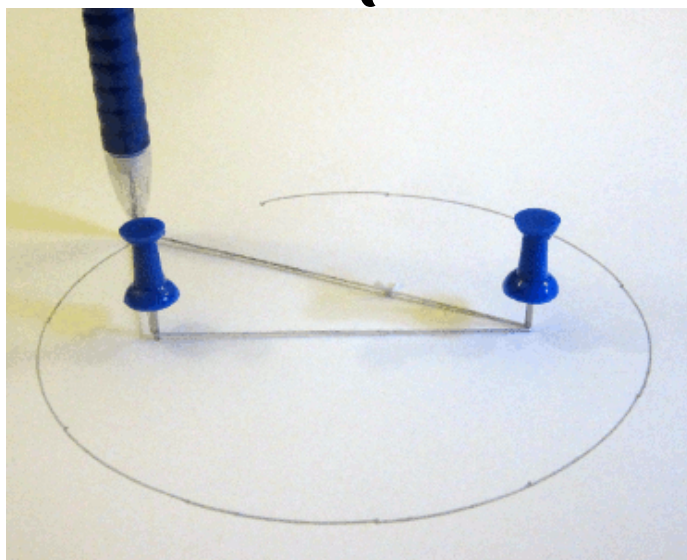
center  $(2, -6)$   
radius  $\sqrt{18} \approx 4.2$



### About Me

1. What matters to you more than anything?
2. What do you make fun of or complain about me behind or in front of my back? Be honest :)

## 11.2 Ellipses

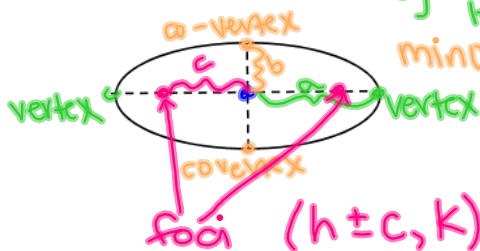


## 11.2 Ellipse Notes

Standard form of an ellipse:  $c^2 = a^2 - b^2$ ,  $a > b > 0$ 

Horizontal (fat)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



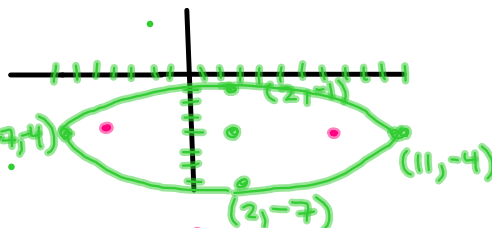
Ex. 1 Write in standard form, list foci and graph

A.  $(x-2)^2 + 9(y+4)^2 - 81 = 0$

$$\frac{(x-2)^2}{81} + \frac{9(y+4)^2}{81} = \frac{81}{81}$$

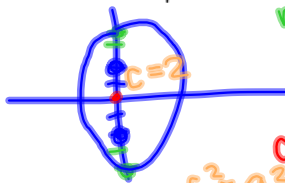
$$\frac{(x-2)^2}{81} + \frac{(y+4)^2}{9} = 1$$

Center  $(2, -4)$   
 $a^2 = 81$   $b^2 = 9$   
 $a = 9$  along  $x$   $b = 3$  along  $y$



$$c^2 = 81 - 9$$

$$\sqrt{c^2} = \sqrt{72}$$

Ex. 2 Write the equation of an ellipse with a major axis of 8 and foci at  $(0, 2)$  and  $(0, -2)$ major axis  $= 2a$ 

$8 = 2a$

$a = 4$

center  $(0, 0)$ 

$$c^2 = a^2 - b^2$$

$$2^2 = 4^2 - b^2$$

$$4 = 16 - b^2$$

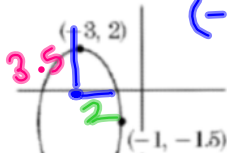
$$-12 = -b^2$$

$$+12 = +b^2$$

foci are on major axis

$$\frac{x^2}{12} + \frac{y^2}{4^2} = 1$$

Ex. 3 Write the equation of the ellipse

center  $(-3, -1.5)$ 

$$\frac{(x+3)^2}{2^2} + \frac{(y+1.5)^2}{(3.5)^2} = 1$$

Ex. 4: Find the foci of an ellipse with vertices

 $(2, 1)$  and  $(-6, 1)$  and a minor axis of length 4

$2b = 4$

$b = 2$

center  $(-2, 1)$ 

$$\text{major } 2a = 8$$

$$a = 4$$

$$\frac{(x+2)^2}{16} + \frac{(y-1)^2}{4} = 1$$

$$B. \quad 4x^2 + 9y^2 - 48x + 72y + 144 = 0$$

$$4x^2 - 48x + 9y^2 + 72y = -144$$

$$4(x^2 - 12x + 36) + 9(y^2 + 8y + 16) = -144$$

$$\left(-\frac{12}{2}\right)^2 = (-6)^2$$

$$\left(\frac{8}{2}\right)^2 = (4)^2$$

$$+ 4(36) + 9(16)$$

$$\frac{4(x-6)^2}{144} + \frac{9(y+4)^2}{144} = \frac{144}{144}$$

$$\boxed{\frac{(x-6)^2}{36} + \frac{(y+4)^2}{16} = 1}$$

center (6, -4)  
hz (fat)

complete  
the square  
TWICE

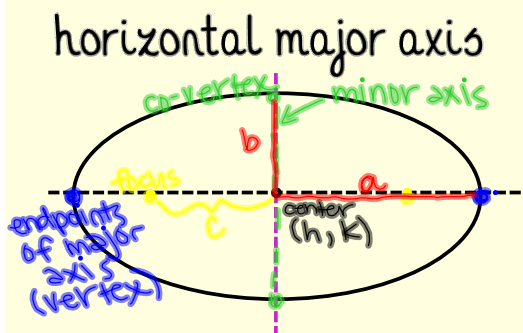
## 11.2 Ellipses

### Practice problems (on desmos)

Work through the desmos problems (link on google classroom). This is your classwork – what you don't finish needs to be done for homework.

# Ellipses

definition: Set of all points P in a plane such that the sum of the distances from P to two fixed points  $F_1$  and  $F_2$ , called the foci, is constant.



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

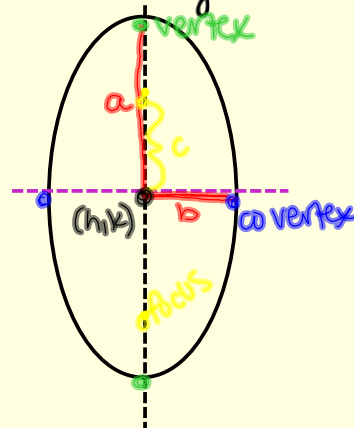
Center:  $(h, k)$

Major Axis: horizontal, length  $2a$

Minor Axis: vertical, length  $2b$

Foci: along the horizontal axis,  $c$  units from the center

vertical major axis



$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

Center:  $(h, k)$

Major Axis: vertical, length  $2a$

Minor Axis: horizontal, length  $2b$

Foci: along the horizontal axis,  $c$  units from the center

$a^2$  is always the bigger number!

Use  $c^2 = a^2 - b^2$   
to find the foci

examples

1.  $\frac{x^2}{9} + \frac{y^2}{36} = 1$  ✓

center  $(0, 0)$

$c^2 = a^2 - b^2$

$c^2 = 36 - 9 = 27 = 3^2 \cdot 3$

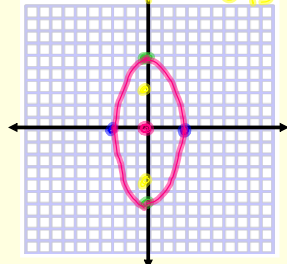
Vertical

$a^2 = 36$

$b^2 = 9$

$a = 6$

$b = 3$



2.  $\frac{(x-1)^2}{25} + \frac{(y+1)^2}{9} = 1$

center  $(1, -1)$

$c^2 = 25 - 9$

$c^2 = 16 \Rightarrow c = 4$

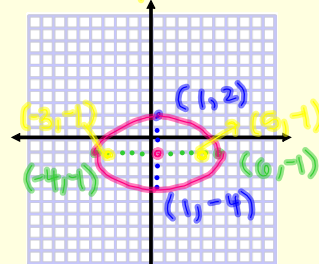
Horizontal

$a^2 = 25$

$b^2 = 9$

$a = 5$

$b = 3$



# ELLIPSES

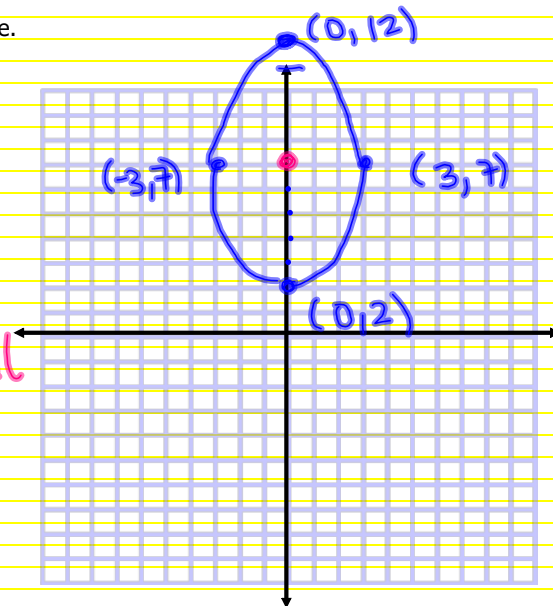
Find the center, a, b, and c, then graph the ellipse.

1. NOT in standard ☹️

$$\frac{25x^2}{225} + \frac{9(y-7)^2}{225} = \frac{225}{225}$$

$$\frac{x^2}{9} + \frac{(y-7)^2}{25} = 1$$

Center (0, 7) Vertical  
 $a = 5$   
 $b = 3$



## GRAPHING TIPS

- Put equation in standard form.
  - Find the center.
  - Determine if the major axis is horizontal or vertical.
  - Find a - the length of the major axis is  $2a$  - and plot the vertices.
  - Find b - the length of the minor axis is  $2b$  - and plot the covertices.
  - Connect the vertices and covertices with a smooth curve.
- If given the focus, remember:  
 $c^2 = a^2 - b^2$

## ECCENTRICITY

measure of the ovalness of an ellipse

$$\frac{c}{a} = e \quad \text{If } \frac{c}{a} \text{ is closer to 0:}$$

Then, the foci are closer to the center.  
 The Ellipse is more circular.

$$\frac{c}{a} = e \quad \text{If } \frac{c}{a} \text{ is closer to 1:}$$

Then, the foci are closer to vertices.  
 The Ellipse is very elongated.

## EXAMPLES

Find a and c. Find the eccentricity, then describe the ellipse.

1.  $\frac{x^2}{16} + \frac{y^2}{25} = 1$

$$b^2 = 16 \quad a^2 = 25$$

$$c^2 = 25 - 16 = 9$$

$$c = 3$$

$$e = \frac{3}{5}$$

close to 1, elongated

2.  $\frac{x^2}{1} + \frac{y^2}{8} = 1$

$$c^2 = 8 - 1 = 7$$

$$c = \sqrt{7}$$

$$e = \frac{\sqrt{7}}{\sqrt{8}} \approx .9...$$

close to 1, elongated

#1-5 ALL, #6-17 EVENS

**Student Practice – Ellipse Day 1**

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

Identify each equation as a circle or an ellipse. If it is an ellipse draw the graph and label the center, vertices, co-vertices. State the domain and range.

1.  $x^2 + y^2 = 81$

2.  $9x^2 + y^2 = 144$

3.  $3x^2 + 3y^2 = 21$

4.  $\frac{(x+2)^2}{9} + \frac{(y+3)^2}{16} = 1$

5.  $\frac{(x+1)^2}{4} + \frac{(y-1)^2}{1} = 1$

Find the foci and vertices of each ellipse:

6.  $\frac{x^2}{25} + \frac{y^2}{16} = 1$

7.  $\frac{(y-5)^2}{169} + \frac{(x+1)^2}{144} = 1$

8.  $\frac{y^2}{36} + \frac{(x+3)^2}{4} = 1$

9.  $9(y-7)^2 + 25x^2 = 225$

10.  $49x^2 + 16y^2 = 784$

11.  $9(x-3)^2 + 81(y+3)^2 - 729 = 0$

Write the equation in standard form.

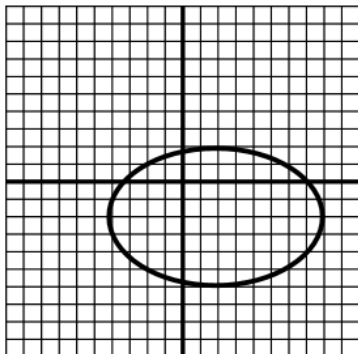
12.  $49x^2 + 64y^2 - 3136 = 0$

13.  $4(x - \frac{1}{2})^2 + 9(y + \frac{2}{5})^2 - 72 = 0$

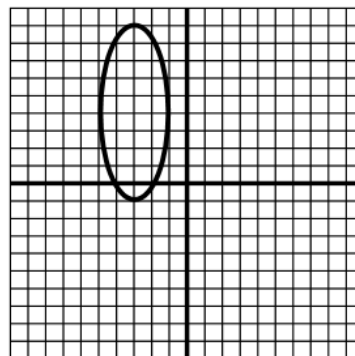


Write the equation in standard form.

14.



15.



16. Statuary Hall is an elliptical room in the United States Capitol in Washington, D.C. The room is 46 feet wide and 96 feet long. Because of a reflective property of an ellipse, a person standing at one focus can hear even a whisper spoken by a person standing at the other focus. (John Quincy Adams is said to have used this feature of the room to overhear conversations.)

A) Find an equation of the ellipse.

B) How far apart are the two foci?



17. An elliptically shaped garden is surrounded on all sides by a wooden walkway. The garden is 15 meters long and 8 meters wide. The walkway is 2 meters wide.

A) Find the equation describing only the garden.

B) Find the equation describing the garden and walkway together.

C) Find the area of just the walkway ( $A = \pi ab$ ).