10.7 Applications Day 1

Test WEDNESDAY 4/25, extra credit due FRIDAY 4/27- no exceptions

Warm-Up Wednesday

Solve for x, show all work!!

1. $3^{x-1} = \left(\sqrt{3}\right)^{x+1}$

$3^{x-1} = \left(3^{\frac{1}{2}}\right)^{x+1}$

$\cancel{3}^{x-1} = \cancel{3}^{\frac{1}{2}(x+1)}$

$\overset{+1}{1x-1} = \frac{1}{2}x + \overset{+1}{\frac{1}{2}}$

$\frac{.5x}{.5} = \frac{1.5}{.5}$

$\boxed{3}$

2. $\log_3 x = \log_3 6 + (\log_3 10 - \log_3 5)$

$\frac{10}{5}$

$\log_3 x = \log_3 6 + \log_3 2$

$\cancel{\log_3} x = \cancel{\log_3} 12$

$\boxed{x = 12}$

About Me

1. Would you rather never have to work again or never have to sleep again (you won't feel tired or suffer negative health effects)?
2. Would you rather have a completely automated home or a self-driving car?

10.7 Notes: Applications of Exponential and Logarithmic Functions

$y_t = y_0 e^{rt}$

Growth and Decay Applications

Formula: $A = Pe^{rt}$ ← time

ending amount → A

starting amount (principal) → P

rate (decimal) → r

Where:

P = initial amount
A = amount at time t
e^{rt} = rate of growth or decay (dependent on time)
r = exponential growth or decay constant
t = time

Note: The half-life of a radioactive substance is the time it takes for half of the substance to decay (disappear).

Example 1: A certain radioactive substance has a half-life of 248 years. How much of a 10 gram sample will remain after 150 years?

① Find r (rate of decay)

$A = Pe^{rt}$

$A = 10/2 = 5$
$P = 10$
$r = ?$
$t = 248$ years

$\frac{5}{10} = \frac{10e^{r(248)}}{10}$

$0.5 = e^{248r}$

$\frac{\ln(0.5)}{248} = \frac{248r}{248}$

$r \approx -0.003...$

decay → negative r

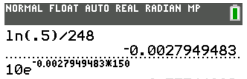
② Use other info to answer ?

$\textcircled{A} = Pe^{rt}$

$= 10e^{r(150)}$

$P = 10g$
$A = ?$
$r = -0.003...$
$t = 150$ years

use copy/paste in calc.



```
NORMAL FLOAT AUTO REAL RADIAN MP
ln(.5)/248
                    -0.0027949483
10e^-0.0027949483*150
                       6.57544889
```

$\boxed{6.575g}$

Example 2: The number of bacteria in a certain culture has an initial count of 500. If the number of bacteria triples every 3 hours, how long will it take to reach a count of 2000?

① Find rate

$A = Pe^{rt}$

$A = 3(500) = 1500$
$P = 500$
$r = ?$
$t = 3$ hrs

ln → $\log_e$

$\frac{1500}{500} = \frac{500e^{r(3)}}{500}$

$3 = e^{3r}$

$\frac{\ln(3)}{3} = \frac{3r}{3}$

$r = 0.366...$

positive (growth)

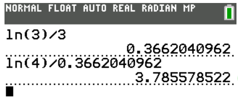
② Find time

$A = 2000$
$P = 500$
$r = 0.366...$
$t = ?$

$\frac{2000}{500} = \frac{500e^{rt}}{500}$

$4 = e^{rt}$

$\frac{\ln 4}{r} = \frac{rt}{r}$



```
NORMAL FLOAT AUTO REAL RADIAN MP
ln(3)/3
                     0.3662040962
ln(4)/0.3662040962
                      3.785578522
```

$\boxed{3.786 \text{ hours}}$

Banking Applications

Tomorrow!!

When w[illegible]g [illegible]nk[illegible]m, [illegible]k [illegible]f th[illegible]t rate [illegible]ounded continuou[illegible]ly, [illegible] at [illegible] s[illegible]eci[illegible] in[illegible]l. [illegible] will te[illegible] you w[illegible]ich [illegible]se[illegible]

If the interest is compounded continuously use:

$$A = Pe^{rt}$$

If the interest is compounded after a specific interval, use: $A = P\left(1+\frac{r}{n}\right)^{nt}$

A = amount after t years
P = principal or inital amount
r = interest rate, written as a decimal
n = number of times compounded in 1 year
t = number of years

Example 3: Sam is investing $800 into an account that earns 3% annual interest.

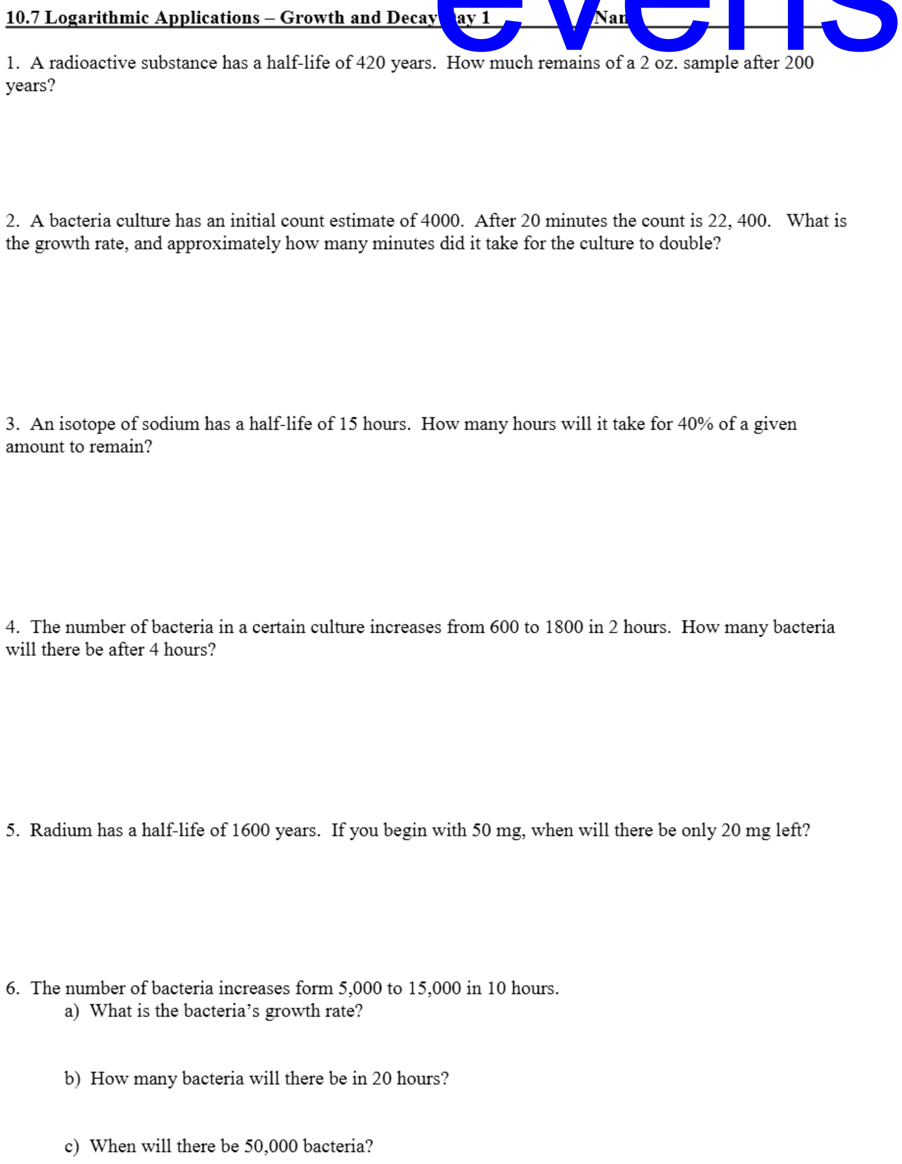
a) If the interest is compounded monthly, how much would he have after 5 years?

b) If the interest is compounded continuously, how much would he have after 5 years?

Example 4: Mrs. Scott has $3500 invested in an account at 5% annual interest rate compounded monthly. How long will it take for her account to reach $5000?

Example 5: Mary invested $2000 in an account 7 years ago. If the interest was compounded quarterly and the money has doubled in that time, what was the interest rate rounded to one decimal?

evens



10.7 Logarithmic Applications – Growth and Decay Day 1 Name

1. A radioactive substance has a half-life of 420 years. How much remains of a 2 oz. sample after 200 years?

2. A bacteria culture has an initial count estimate of 4000. After 20 minutes the count is 22, 400. What is the growth rate, and approximately how many minutes did it take for the culture to double?

3. An isotope of sodium has a half-life of 15 hours. How many hours will it take for 40% of a given amount to remain?

4. The number of bacteria in a certain culture increases from 600 to 1800 in 2 hours. How many bacteria will there be after 4 hours?

5. Radium has a half-life of 1600 years. If you begin with 50 mg, when will there be only 20 mg left?

6. The number of bacteria increases form 5,000 to 15,000 in 10 hours.
   a) What is the bacteria's growth rate?

   b) How many bacteria will there be in 20 hours?

   c) When will there be 50,000 bacteria?

7. The polonium isotope ^{210}Po has a half-life of 140 days. If the initial sample is 20 mg, how much will be left after two weeks?

8. The population of a city increases 5% per year. If the present population is 500,000, what will the population be in 10 years?

9. Find the half-life of a radioactive substance for which a) $k = \frac{1}{100}$, and b) $k = 1000$.

10. If ¼ of a radioactive substance disintegrates in 10 days, what is its half-life?

11. A physicist has 12 grams of radioactive bismuth, which has a half-life of 5 days.
    a) How much will be left after 15 days?

    b) How long will it take for 2 grams to disappear?

12. In 1950 the population of a city was 80,000, and in 1960 it was 100,000.
    a) What is its yearly growth rate?

    b) What is the population in 1980?

    c) What is the population in 2010?

10.7 Notes: Applications of Exponential and Logarithmic Functions

$Y_t = Y_0 e^{rt}$

Growth and Decay Applications

Formula: $A = Pe^{rt}$ ← rate (decimal)

↑ time

ending amount

beginning amount (principle)

Where:

P = initial amount
A = amount at time t
e^{rt} = rate of growth or decay (dependent on time)
r = exponential growth or decay constant
t = time

Note: The half-life of a radioactive substance is the time it takes for half of the substance to decay (disappear).

Example 1: A certain radioactive substance has a half-life of 248 years. How much of a 10 gram sample will remain after 150 years?

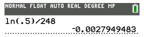
① Find r

$\frac{1}{2} = e^{r(248)}$

$0.5 = e^{248r}$

$\frac{\ln(0.5)}{248} = \frac{248r}{248}$

$r \approx -0.003$



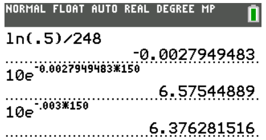
```
NORMAL FLOAT AUTO REAL DEGREE MP
ln(.5)/248
                         -0.0027949483
```

② Use other info

$\textcircled{A} = Pe^{rt}$

$A = 10e^{r(150)}$

$A = \boxed{6.575 g}$



```
NORMAL FLOAT AUTO REAL DEGREE MP
ln(.5)/248
                         -0.0027949483
10e^(-0.0027949483*150)
                            6.57544889
10e^(-.003*150)
                           6.376281516
```

Example 2: The number of bacteria in a certain culture has an initial count of 500. If the number of bacteria triples every 3 hours, how long will it take to reach a count of 2000?

3·500

① Find r.

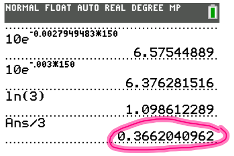
$A = Pe^{rt}$

$\frac{1500}{500} = \frac{500e^{r(3)}}{500}$

$3 = e^{3r}$

$\frac{\ln 3}{3} = \frac{3r}{3}$

$r \approx .366$



```
NORMAL FLOAT AUTO REAL DEGREE MP
10e^(-0.0027949483*150)
                            6.57544889
10e^(-.003*150)
                           6.376281516
ln(3)
                           1.098612289
Ans/3
                          0.3662040962
```

② Find time (t)

$A = Pe^{rt}$

$\frac{2000}{500} = \frac{500e^{(0.366)t}}{500}$

$4 = e^{(0.366)t}$

$\frac{\ln 4}{0.366} = \frac{0.366t}{0.366}$

$\boxed{t = 3.786 \text{ hours}}$